KISS: modeling to understand

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Simple models for long term dynamics

Environment dynamics involve time scales in the order of human generations. Hence constants of economic models, especially at equilibrium, become variables. For instance:

- Agents' preferences
- Agents' knowledge and technology
- Resources

Hence the idea of coupling dynamics, but simplifying the dynamics of each single component.

- Because of our limited knowledge about the world
- Because of our limited intelligence to understand the model and the simulation results.

Heterogeneity and Increasing Returns Drive Socio-economic Transitions

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Let's KISS.

Dynamic of adoption of environment friendly products

"Green"technologies are often more expensive than standard technologies.

- For a given set of products, what distribution of market shares will be observed, depending upon prices and agents preferences?
- Under what conditions could "green" technologies prevail or at least survive?
- Knowing the above information, how could producers or government agencies react?

Let's rush to ABM .... Or first check the result of the KISS approach:

KEEP IT SIMPLE STUPID !

A Simple Set of Assumptions:

- Environmentally superior technologies are more expensive (given a similar market share).
- Products with a larger market share are cheaper and/or more attractive (increasing return to scale).
- Buyers vary in their extent to which they want to pay for environmental benefits (heterogeneity of willingness to pay, WTP).

The model: increasing returns and willingness to pay.

Three technological options: standard (0), intermediate (1), "green" (2).

- Each option i has its own maximum cost P_{0i}
- All options have the same returns to scale k (e.g., $p_i = P_{0i} - ku_i$)
- Distribution of willingness to pay for environmental benefits (e.g., uniform, bell-shaped)

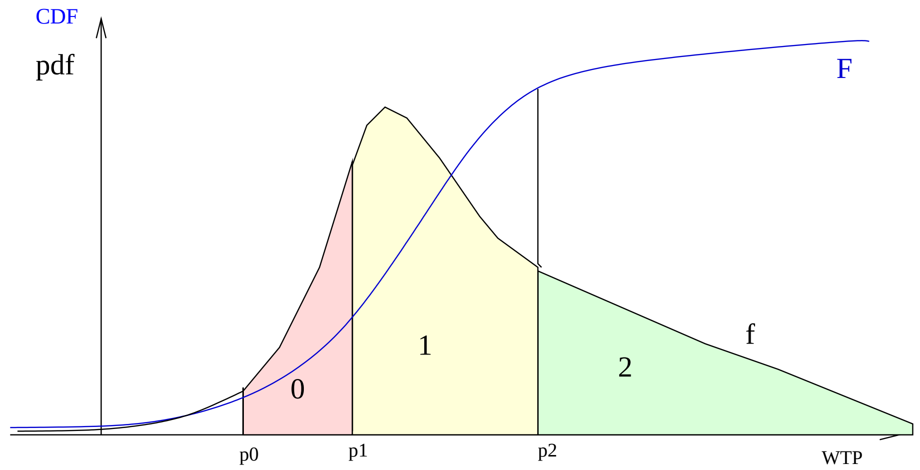


Figure 1: Distributions of Willingness to Pay (WTP). Customers with WTP larger than p_i may choose car i with market share u_i equal to colored area.

Equations

$$p_i = P_{0i} - k \times u_i \tag{1}$$

The price of a product follows a linear return to scale function.

Because of the role of market share in fixing actual prices, the order among actual prices p_i may differ from maximum prices. First operation: to order products according to actual prices. A product j with price p_j larger than price p_{j+1} of a product with higher environmental quality, disappears from the market.

$$u_i = F(p_{j+1}) - F(p_j) \tag{2}$$

where F(p) is the cumulative WTP distribution.

The above equations of market shares at equilibrium are thus soluble. The dynamics of market shares is given by:

$$u_j(t+1) = (1-\lambda) \times u_j(t) + \lambda \times (F(p_{j+1}, t) - F(p_j, t)) \tag{3}$$

Only a fractionλ of the agents renew their appliance at each time step.

WTP distributions

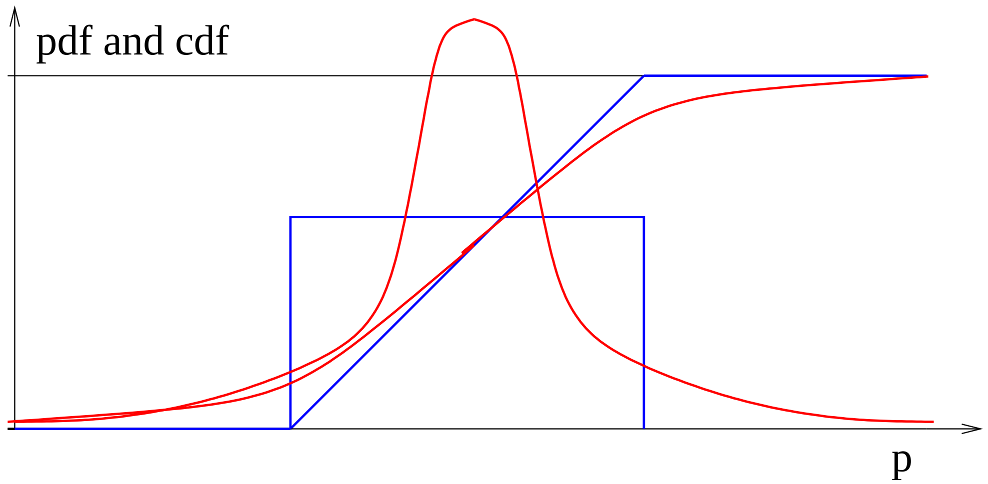
Uniform distribution, with a cumulative distribution $F(p)$:

$$F(p) = \frac{p - p_m}{p_M - p_m} \tag{4}$$

For the bell-shaped distribution the cumulative distribution is a logit expression:

$$F(p) = \frac{1}{1 + exp(-\beta p)} \tag{5}$$

where β is inversely proportional to the width of the distribution.



Exact results

The equation for the attractor of u_2 is :

$$u_2 = 1 - F(p_2 - k u_2) \tag{6}$$

For the uniform WTP distribution:

$$u_2 = \frac{p_M - p_2}{p_M - p_m - k} \tag{7}$$

(valid under certain conditions!)

The market share of the green car only depends upon two reduced parameters, $\frac{p_M - p_2}{p_M - p_m}$ and $\frac{k}{p_M - p_m}$.

It increases when $p2$ decreases and when k increases (increasing returns to scale).

The same results hold for the bell-shaped WTP distribution.

Simulation results, Time evolution of market shares

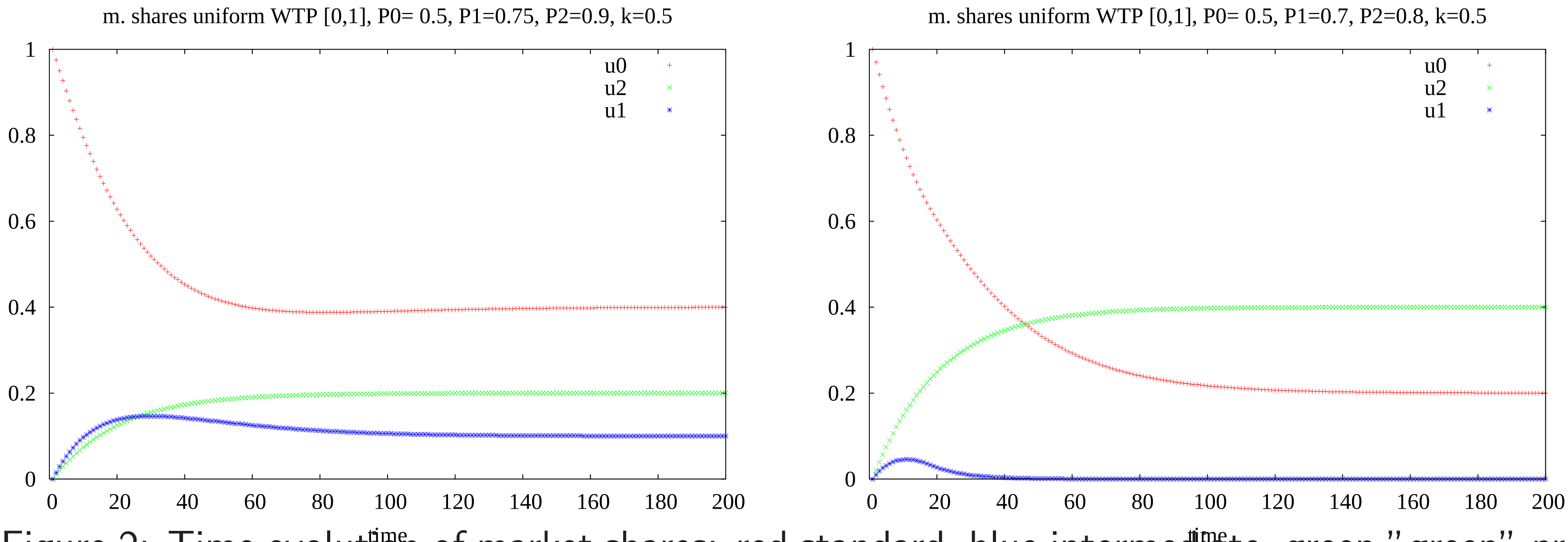


Figure 2: Time evolution of market shares: red standard, blue intermediate, green "green" products. $(\lambda)^{-1} = 10$.

Simulation results, Dynamical regimes

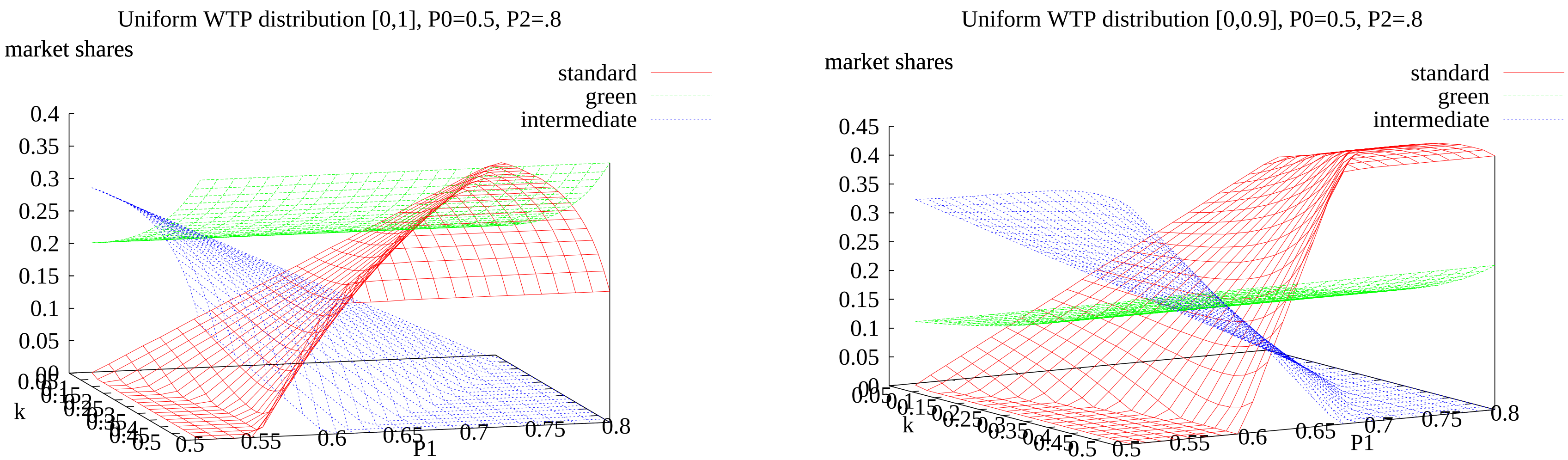


Figure 3: Asymptotic market shares as of function of k and P_{01}. Left: Square WTP distribution [0,1], $P_{00} = 0.5$, $P_{02} = 0.8$. P_{01} varies between P_{00} and P_{02}. The green sheet correspond to u_2 the market share of the green product, while the blue and red sheets respectively correspond the market shares of the intermediate and standard products. Right: Square WTP distribution [0,1.1].

Hysteresis

Equation:

$$u = 1 - F(p - ku) \tag{8}$$

is known to produce 'phase' or 'regime' transitions when the number of solutions goes from one to three as a function of a parameter which is k/w in our case; w is the widthe of the WTP distribution. (e.g. Mean Field theory of ferromagnetism, socio- economics of increasing returns or social influence:

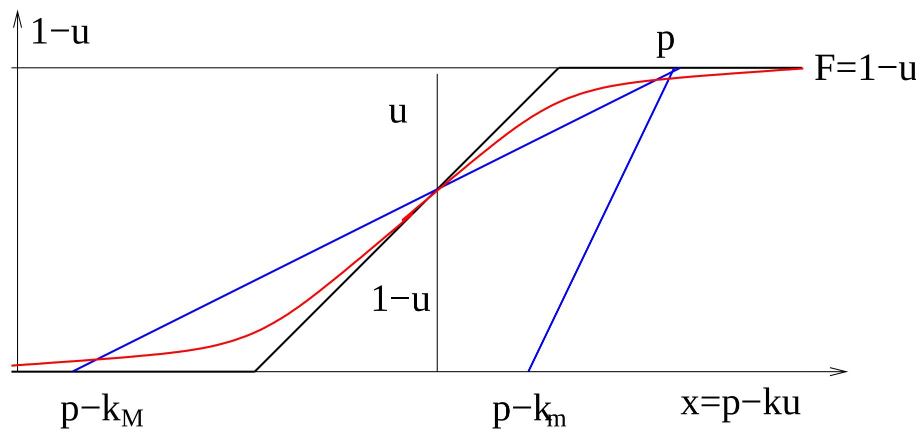


Figure 4: Graphical solution of equation (4) rewritten as $1 - u = F(p - ku)$. Abscissa is $x = p - ku$, and ordinate $1 - u$. The $F(x)$ curve (cdf), represented in red for bell shaped distribution of WTP and in black for a uniform distribution, intersects blue lines 1-u in one or three points according to the value of k for the particular choice of p. $k_m \leq 1 \leq p_M$.

Hysteresis

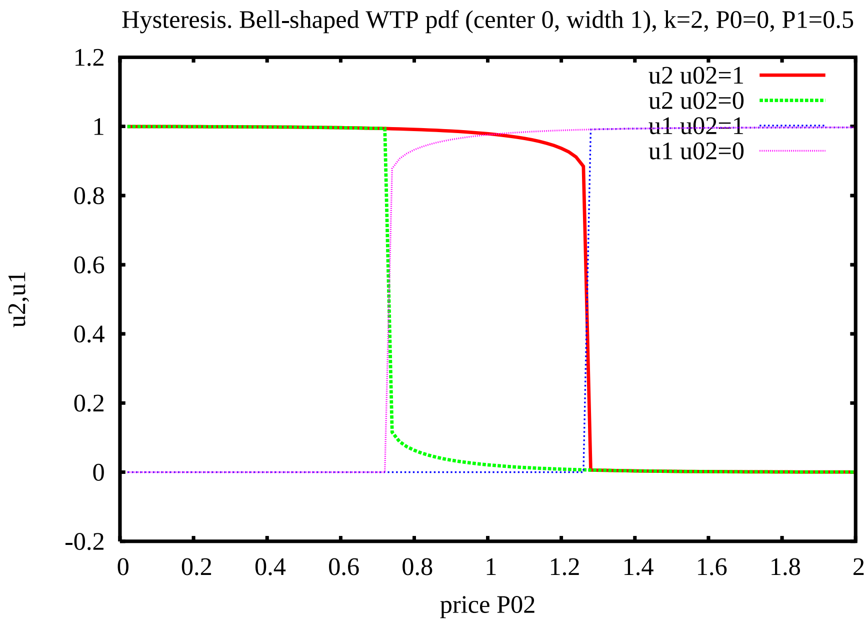


Figure 5: A hysteresis cycle obtained for u_2 when P_{02} is varied. The red curve is u_2 for $u_2(0) = 1$ and the green curve when $u_2(0) = 1$. The two curves coincide when the price P_{02} is either small($u_2(0) \simeq 1$) or large ($u_2(0) \simeq 1$). They differ in the intermediate price region.

Which of the two other products dominate or how they share the market when $u_2 = 0$ depends upon the actual values of P_{00} and P_{01}.

Why should we care about hysteresis?

Since there are several attractors in this regime, the issue of the adoption regime is especially sensitive to the parameter set-up: if we now consider that parameters can be under the influence of decision makers such as producers or government agencies, or exogenous events (e.g. oil prices), the hysteresis regime can bring huge consequences for small parameter changes. For instance:

- If price P_{02} is lowered under the action of producers or government subsidies, a transition from the $u_2 \simeq 0$ attractor to the $u_2 \simeq 1$ attractor can be induced.
- Such an action does not have to be permanent: it might suffice to bring u_2 above the separatrix, the central fix point, to bring the system in the basin of attraction of high u_2.
- Competitors might also have equivalent strategies.
- Sharp transition can also be induced in this region by advertising by decision makers, thus changing the WTP distribution.

Multiple attractors are a challenge for scientists, but they are opportunities for decision makers. (Parameter changes in the single attractor regime also influence the outcome of the dynamics, but their influence is far less dramatic).

Conclusions: Why KISS?

- Axelrod: limited cognitive capacity of scientists, bugs!
- Because you have a limited knowledge about the inputs of the model, elementary processes and parameters, you study regime diagrams and search for generic properties rather than try try to predict precise values of the output.
- Better insight!
- When do you need ABM? In our problem, we would e.g. consider using ABM if we wanted to take into account cognitive factors such as learning (individual or societal), power issues, endogenous societal changes etc.

Transitions on social nets

What happens when we add a social network to heterogeneity of buyers + increasing returns?

The "direct" approach is to start from the social network, randomly attribute to each node i an idiosyncratic utility U_i and suppose that a node with neighbours j will buy (and take $S_j = 1$) when

$$U_i + \sum_{j=1}^{n} k S_j > p \tag{9}$$

Where p is the price of the good, k the social influence coefficient, and S_j is the choice of agent j, equal to one if she bought or 0 if she did not.

This model is equivalent to a ferromagnet in random field.

Transitions on social nets

Predictions:

- According to the width of the utility distribution with respect to the intensity of the social influence $k.n$, where is n is the number of neighbours, two dynamical regimes are observed: continuous vs first order transitions.
- Hysteresis is observed when the price is varied across the first order phase transition.

Visualisation of the 2d INCA display avalanches of acquisition of the green product under favourable price conditions. They can sweep the whole lattice for strong social influence coefficient, or remain limited to "islands" on the network. The size of the islands, larger than one, reflects the local correlation in purchase (coarsening dynamics).

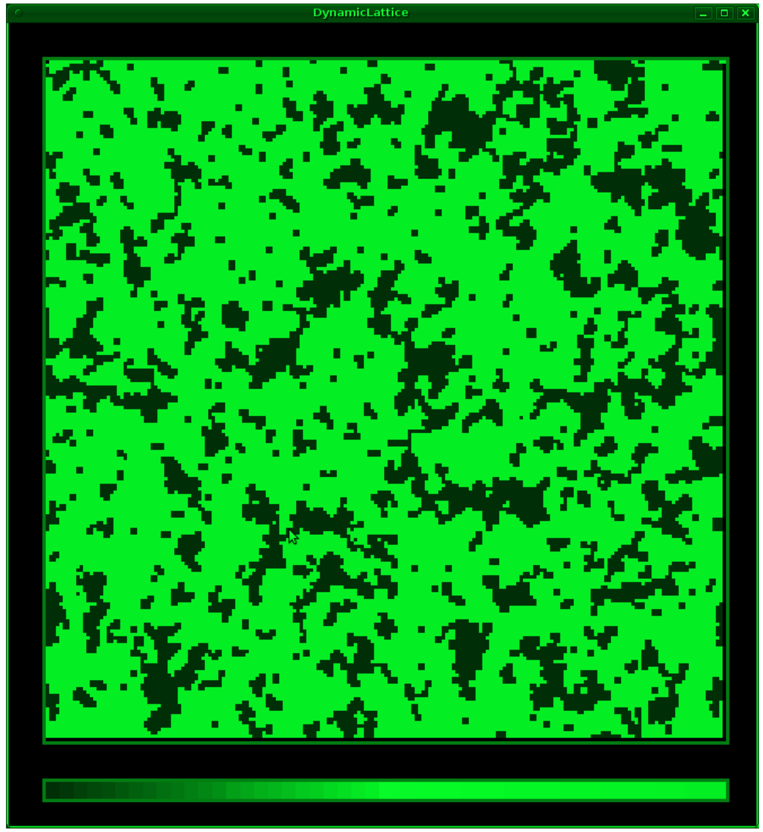


Figure 6: Lattice model of green car purchase. Green sites purchased green cars. Bell-Shaped distribution function of WTP. Social influence factor $k = 0.45$.

Simulation program and parameters

For bell-shaped distributions:

$$\theta(j) = 1/(1 + exp((-ran2(seed) + 0.5) * w))$$

if social influence $\geq$ WTP-price

if $\quad (\Sigma * \kappa.ge.\theta(i) + p) \qquad$ then $\quad nspin(i) = 1$

$p = -0.0010088, w = 13.8, \kappa = 0.25, L = 149$ Gives the verge of the transition. p is then decreased by 1 perc.

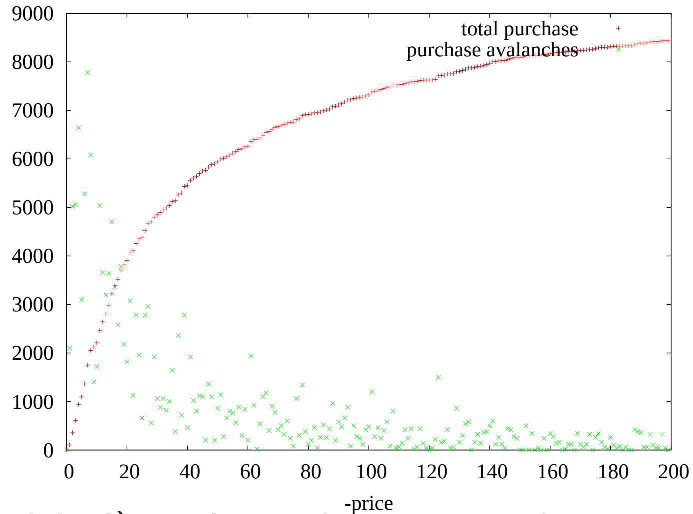


Figure 7: Total purchase (red '+') and purchase avalachances (green x) when price is decreased. Bell-Shaped distribution function of WTP. Social influence factor $k = 0.45$.