Elements from Evolutionary Dynamics

for Elementary Sustainable Economy Models

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Outline

Sustainable Economy

A biologist's perspective

Sustainability $\Rightarrow$ Linking economic processes to environmental processes (biotic & abiotic)

Economy Rationality & anticipation: Strategic behaviour

Biology Mutation (random) & selection: Feedback with environment

Sustainability $\approx$ Persistence

Ecology and evolution

Subjects, processes

Nothing in biology makes sense except in the light of evolution

Dobzhansky

Evolutionary dynamics

$\Updownarrow$

Population dynamics

$\Updownarrow$

Environmental dynamics

The Ecological Theatre and the Evolutionary Play

Hutchinson

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Replicator dynamics

Ingredients

- Reproduction (clonal)
- Mutation
- Selection
- Constant fitness
- Infinitely large, well-mixed population, . . .

Quasispecies equation

Eigen & Schuster

$$\dot{x}_i = \sum_{j=1}^{n} \mu_{ji} f_j x_j - \phi(\mathbf{x}) x_i$$

where

x_i	$\equiv$	frequency of type i
μ_{ji}	$\equiv$	mutation probability of type j to i
f_i	$\equiv$	fitness of type i
$\phi(\mathbf{x})$	$\equiv$	average fitness $\sum_{i=1}^{n} f_i x_i$

Characteristics

Simplex dynamics

- Quadratically nonlinear dynamical system
- Standard eigenvalue problem
- Unique, globally stable equilibrium
- In general not maximizing fitness (mutation-selection balance)

Adaptive dynamics

Ingredients

- Reproduction
- Mutation
- Selection $\longleftarrow$ *interaction between individuals and environment*
- Fitness depends on individual trait value and its environment

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Canonical equation

Dieckmann & Law

$$\frac{d\hat{s}}{dt} = \mu(\hat{s})\,\frac{\sigma_0^2(\hat{s})}{2}\,n(\hat{s})\,\partial_1 f(\hat{s},\hat{s})$$

where	$\hat{s}$	$\equiv$	mean of distribution of trait value s
	$\mu(s)$	$\equiv$	mutation probability
	$\sigma_0^2(s)$	$\equiv$	variance of mutation distribution
	$n(s)$	$\equiv$	equilibrium population size
	$f(s', s)$	$\equiv$	fitness of s'-individuals in s-population

Changing fitness landscapes

- Separation of time scales
- Small mutational steps
- Invasion implies fixation
- *Trait Substitution Sequence* (jump process)

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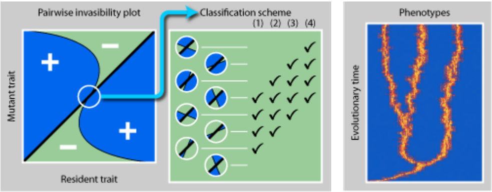

Feedback through environment

Fitness measure

Fitness should be dependent of:

- x The trait being reproduced
- E The environment in which reproduction takes place

Invasion fitness Long-term growth rate
of a rare mutant with trait value x
in an environment set by a resident with trait y

Notation:

$$\sigma(x, E(y))$$

Evolutionary optimization

Environmental feedback

Consider:

- Life-history trait x
- Constant environmental condition E
- Density dependence; unique solution $E = \eta(x)$

Feedback rules:

1. If density dependence reduces life-time offspring production:
   $\hat{x}$ is evolutionarily steady iff $x \mapsto R_0(x, V)$ is maximal for $x = \hat{x}$
2. If density dependence uniformly increases mortality rate:
   $\hat{x}$ is evolutionarily steady iff $x \mapsto r(x, V)$ is maximal for $x = \hat{x}$

Density dependence

One-dimensional E and monotonic feedback rule:

1. If E is one-dimensional
   and $E \mapsto R_0(x, E)$ is increasing (decreasing):
   Then $\hat{x}$ is evolutionarily steady iff
   $x \mapsto \eta(x)$ is minimal (maximal) for $x = \hat{x}$

Interpretation The type that can keep its position under the worst environment cannot be invaded by any other type

Important Optimization approaches are only valid when the eco-evolutionary feedbacks are of a very special kind

Eco-Evo

Economic indicators

Biology	Evolutionary optimization criteria depend on environmental feedback
Economy	Economic indicators: GDP, GNP?

Sustainability indicators shaped by eco-environmental links?

- Different from taxing external costs
- Reverse engineering?
- Which constraints, regulations could yield sustainability optimization?

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References

Sigmund *Games of Life: Explorations in Ecology, Evolution, and Behaviour*. Oxford UP, 1993.

Nowak *Evolutionary Dynamics: Exploring the Equations of Life*. Harvard UP, 2006.

Metz, Mylius & Diekmann "When does evolution optimize? *Evol. Ecol. Res.* 10: 629–654, 2008.

Dercole & Rinaldi *Analysis of Evolutionary Processes: The Adaptive Dynamics Approach and Its Applications.* Princeton UP, 2008.

Dieckmann & Metz (eds.) *Elements of Adaptive Dynamics.* Cambridge UP, 2011.