Consumer choice and interaction models

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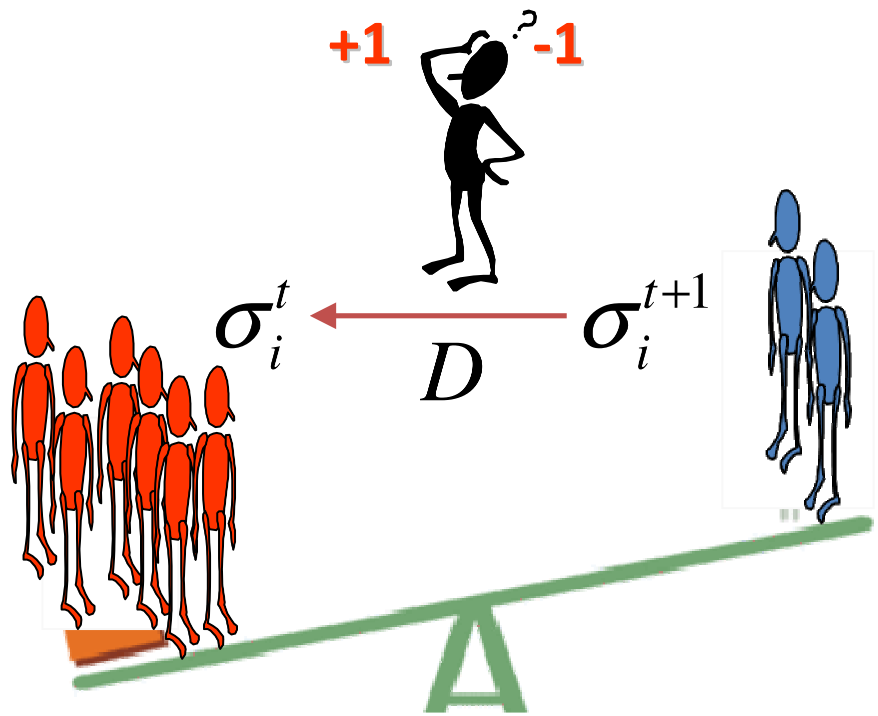
$$\sigma_i^{t+1} \xleftarrow{\quad D \quad} \sigma_i^{t}$$

CRS

Thresholds, Social Impact, and Utility

contesting frames
for modeling human choice



Binary Choice Models

+1  -1

Opinion of i-th individual in the next time step σ_i^{t+1} $\xleftarrow{D}$ σ_i^{t} Opinion of i-th individual in the current time step

Decision Rule

Choices:

$$\sigma_i^{t}, \sigma_i^{t+1} \in \{+1, -1\}$$

$$\sigma_i^{t+1} = D\left(\sigma_i^{t+1}, \sigma_i^{t}, h_i, \mathbf{N_i}, \varepsilon_i\right)$$

$\mathbf{N_i}$ Social environment of i-th individual

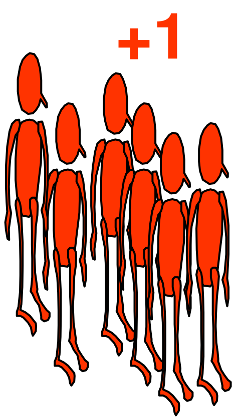
h_i Individual preferences

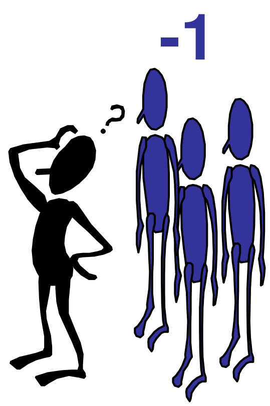
ε_i Random factors affecting i-th individual

Threshold Models

Example: Granovetter model

Decision Rule: $$\sigma_i^{t+1} = \begin{cases} +1 \text{ if } m > \sigma_i^{Th} \\ -1 \text{ if } m < \sigma_i^{Th} \end{cases}$$





$$m \equiv \frac{1}{N} \sum_j \sigma_j$$ **Mean Choice**

N **Number of Agents**

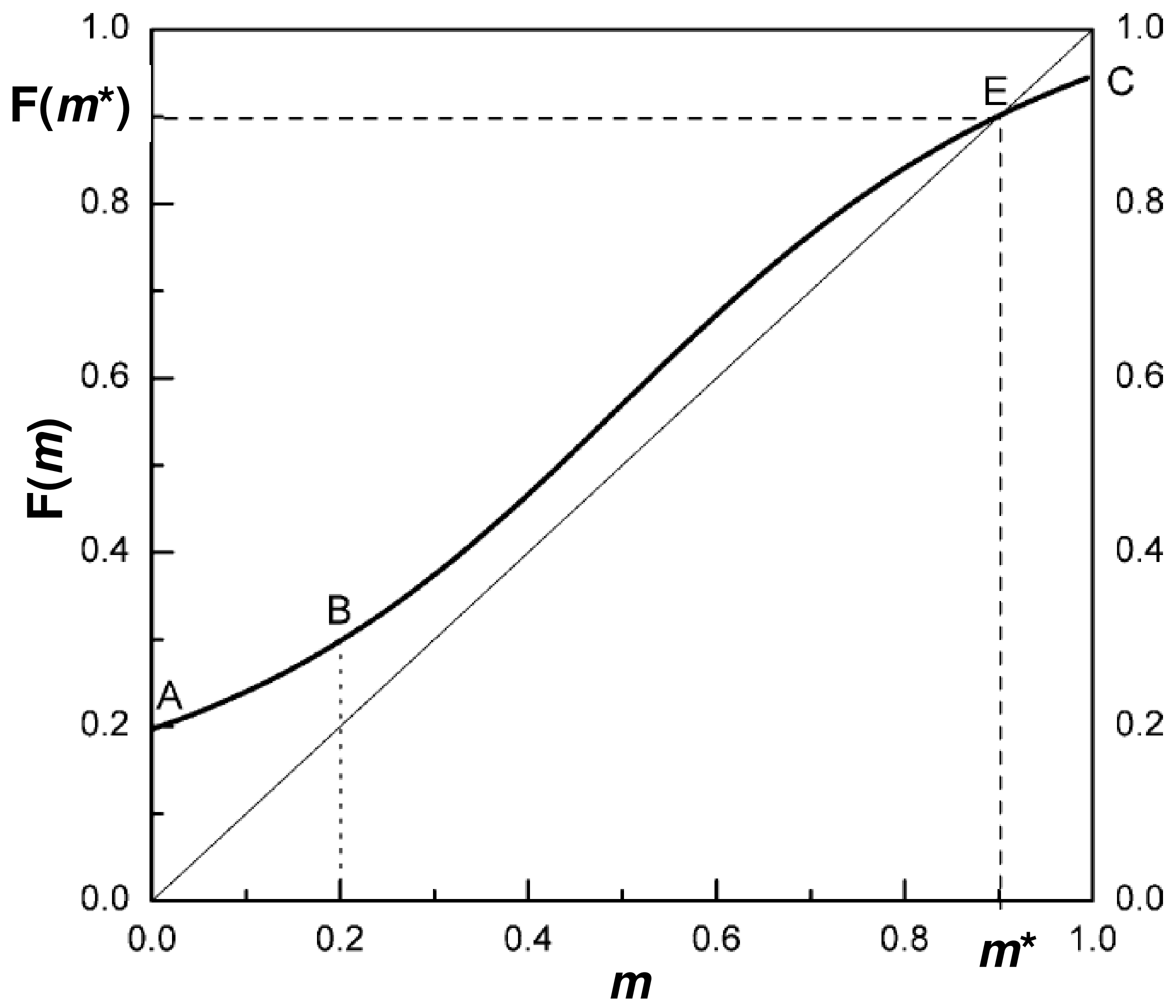
σ_i^{Th} **Thresholds of i-th Agent**

Amount of agents choosing +1 which cause i-th agent to choose +1

Small differences in threshold distributions may lead to radically different aggregate outcomes.

Threshold Models

Example: Granovetter model



Distribution of thresholds

Amount of agents who choose ”+1” for mean choice ***m***

Percentage of population „engaged” (”+1” choice)

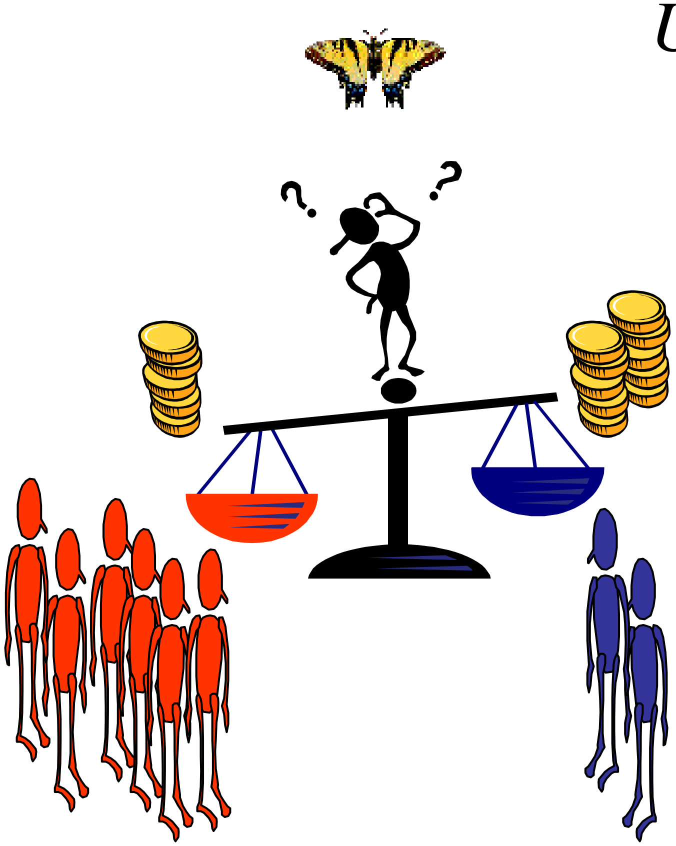
Social Economics Models

Example: Brock-Durlauf Model

Decision Rule: (Utility Function)

$$\sigma_i^{t+1} = \max_{\sigma_i^{t+1} \in \{+1,-1\}} U_i\left(\sigma_i^{t+1}, \mathbf{N_i}, \varepsilon_i\right)$$

$$U_i = \underbrace{h_i \sigma_i'}_{\text{Individual Preferences}} - \underbrace{E_i\left[\sum_{i \neq j} \frac{1}{2} J_{ij} \left(\sigma_i' - \sigma_j\right)^2\right]}_{\text{Social Influence}} + \underbrace{\varepsilon_i(\sigma_i')}_{\text{Random Component}}$$



$$\Pr\left(\varepsilon_i(-1) - \varepsilon_i(+1) \leq z\right) = F_{\beta_i}^{\log}(z) = \frac{1}{1 + e^{-\beta_i z}}$$

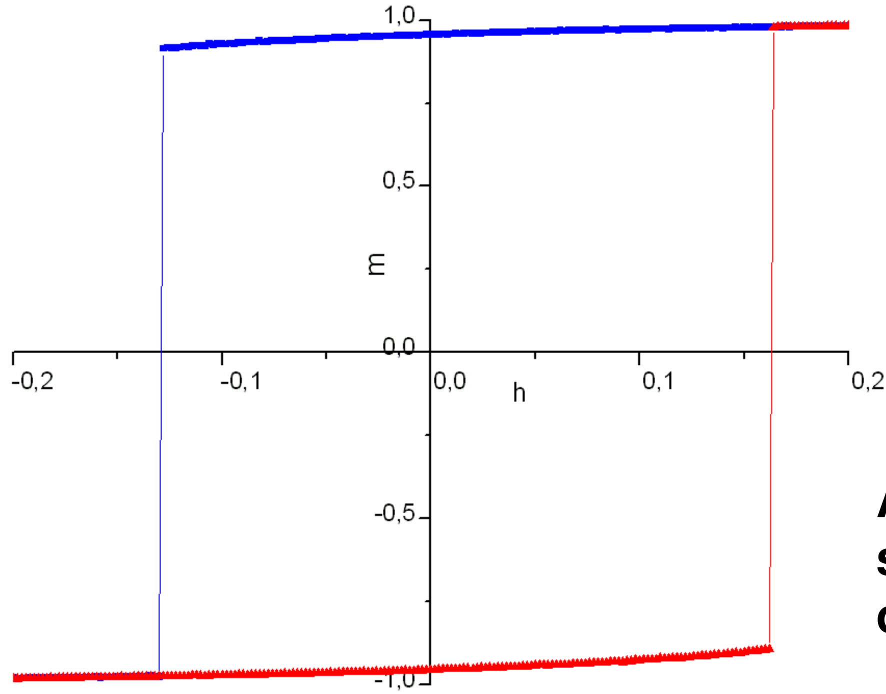
1/β - „social temperature"

Decision Rule:

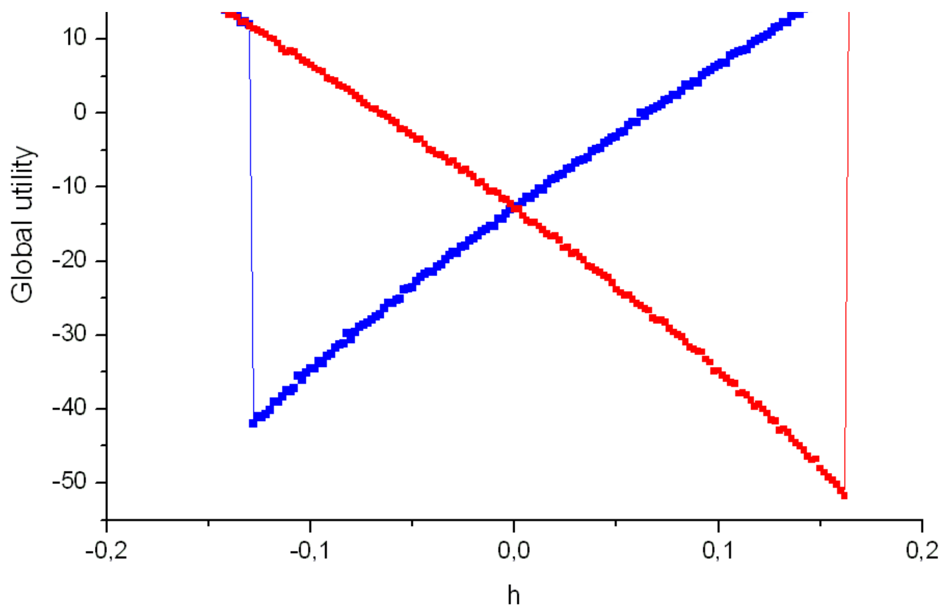
if utility(choice 1) > **utility**(choice -1) **then** choose 1
else choose -1

Multiple equilibria can emerge due to social interactions.

complete pairwise network



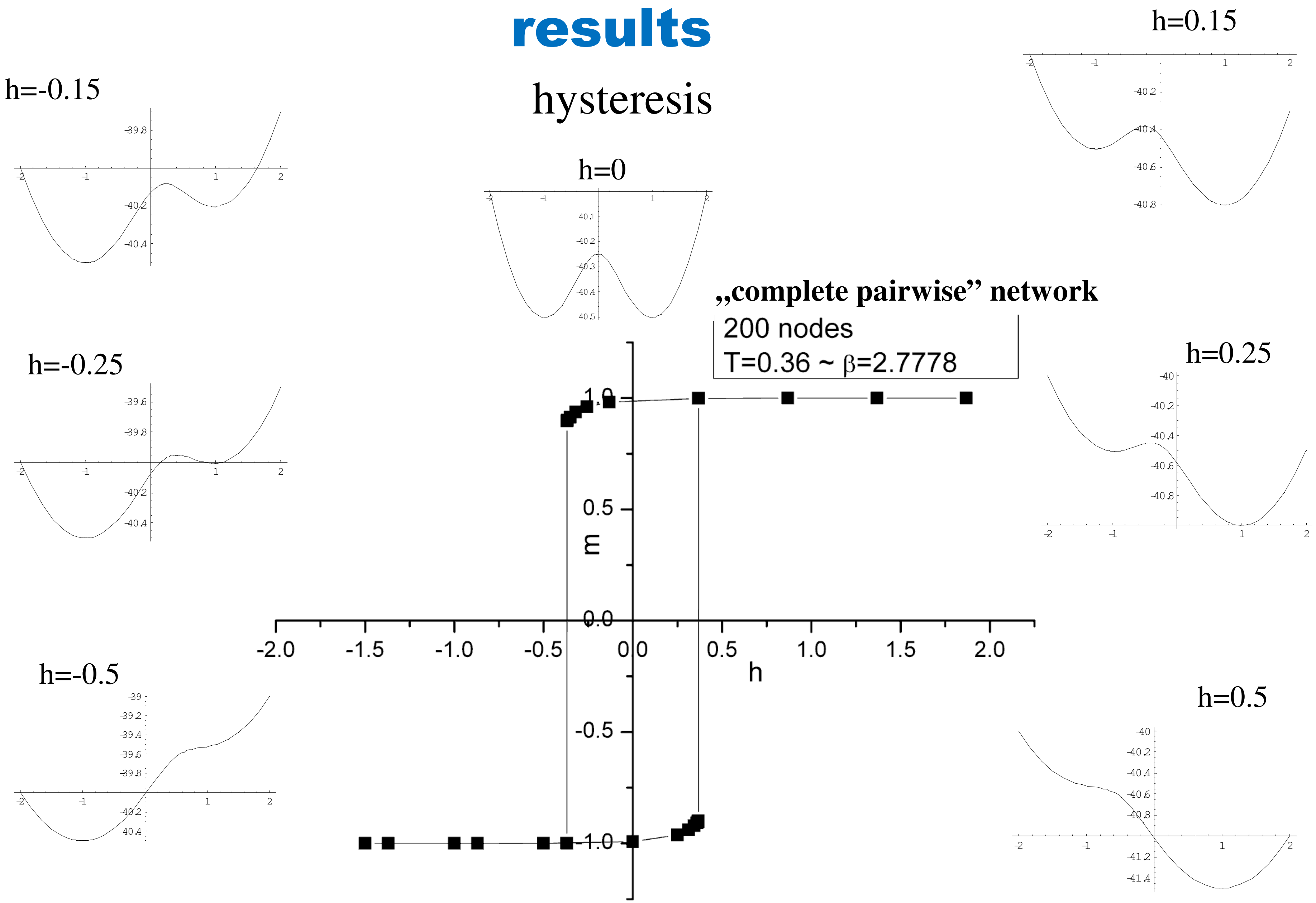
Average choice vs. strength and direction of individual preferences h



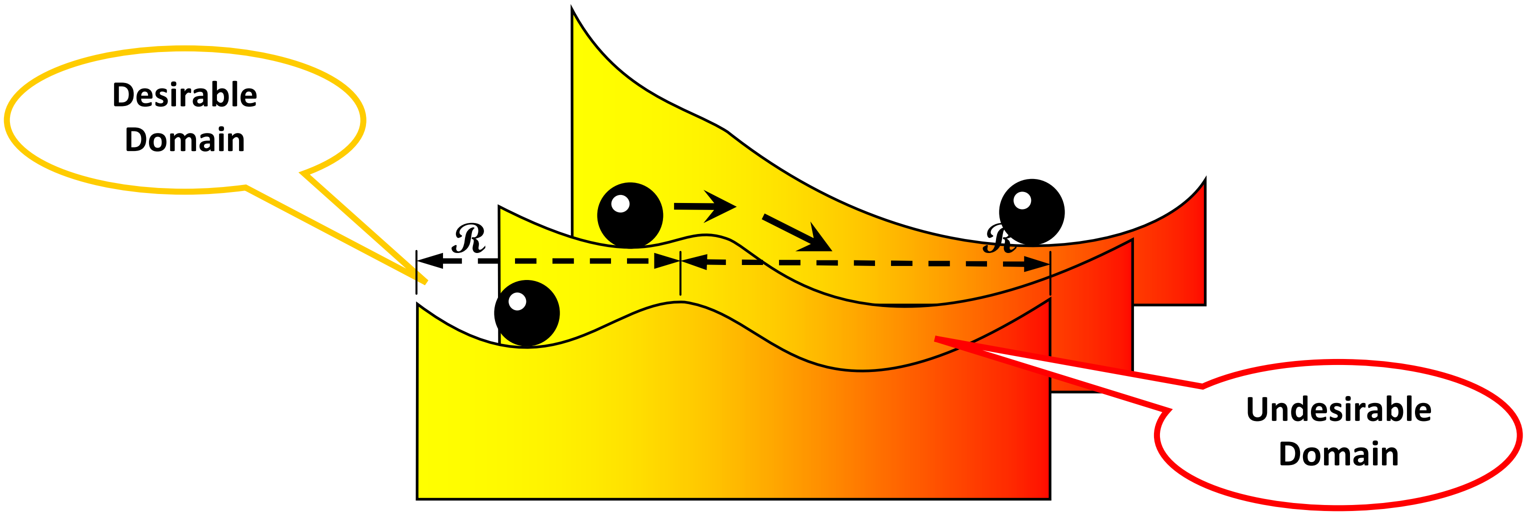
Global utility (deterministic part) strength and direction of individual preferences h

Brock – Durlauf Model: results

hysteresis



Ball & Cup Heuristic



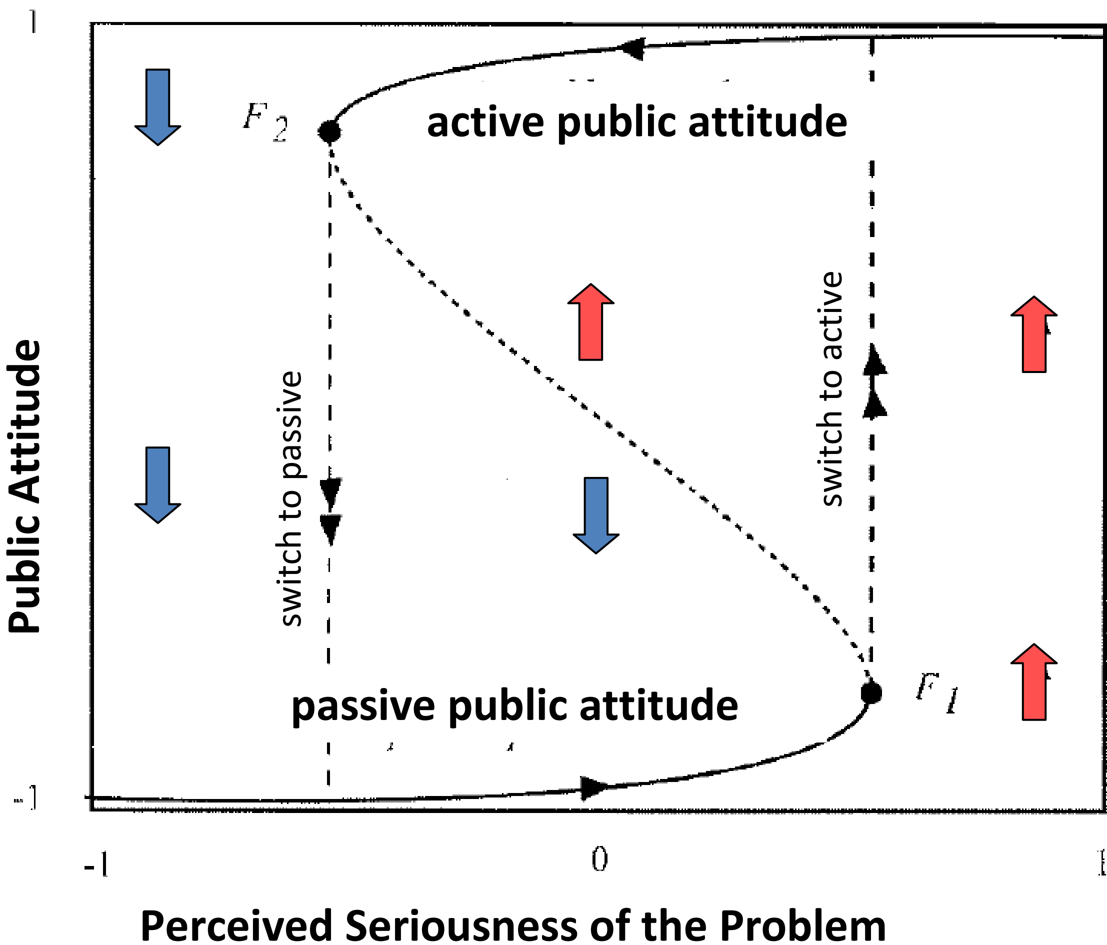
- **Valleys** – desirable and undesirable stability domains

  Domain – set of system states fulfilling certain criteria
- **Balls** – current system state
- **Arrows** – changes in a system state (e.g. caused by disturbances)

Slow Response of Societies to New Problems: Causes and Costs

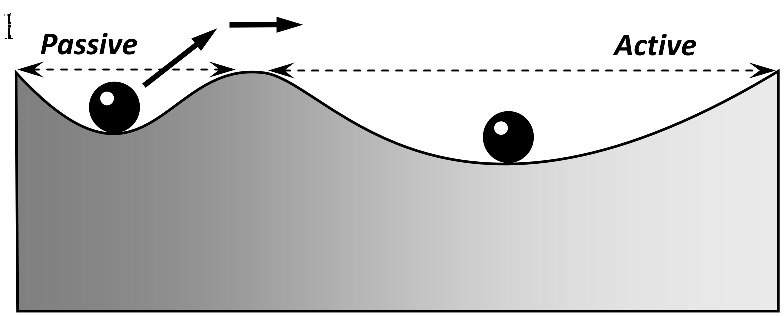
Marten Scheffer, Frances Westley,and William Brock

Ecosystems (2003) 6: 493–502



The degree of hysteresis in public attitude towards the need to regulate a problem is predicted to be larger in situations with:

- high peer pressure
- lack of strong opinion leaders
- complex problems
- relatively homogeneous populations.



Dynamic Social Psychology Models

Example: Nowak – Latane Model

Social Impact I

”Any influence on individual feelings, thoughts, or behavior that is exerted by the real, exerted or imagined presence or actions of others” (Latane’ 81)

$$I = f(S \cdot d \cdot N)$$

Strength	Immediacy	Number
$I \propto S$	$I \propto d^{-2}$	$I \propto \sqrt{N}$

Examples of social impact:

conformism, obedience, imitation, encouragement, stage fear

Dynamic Social Psychology Models

Nowak – Latane Model

Decision Rule: $$\sigma_i^{t+1} = -\operatorname{sgn}\left(\sigma_i^t I_i\right)$$

$$I_i = \sqrt{\sum_j \frac{p_j}{d_{ij}^2}\left(1 - \sigma_i^t \sigma_j^t\right)} - \sqrt{\sum_j \frac{s_j}{d_{ij}^2}\left(1 + \sigma_i^t \sigma_j^t\right)}$$

Impact Function

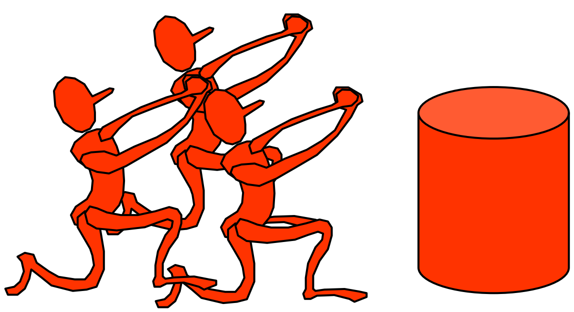
d_{ij} – social distance

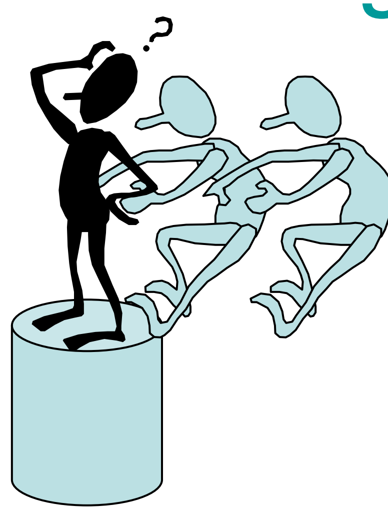
Persuasiveness

p_j – strength of persuasion from j-th agent

Supportiveness

S_j – strength of support from j-th agent





if Persuasiveness > Supportiveness then Change Opinion

Equivalence of models with Utility Function and Impact Function

$$U_i\left(\sigma_i^{t+1},\sigma_i^t,\{\sigma_{j\neq i}\}\right)\equiv -\sigma_i^t\sigma_i^{t+1}I_i\left(\sigma_i^t,\{\sigma_{j\neq i}\}\right)$$

a) IF $I_i\left(\sigma_i^t,\{\sigma_{j\neq i}\}\right)<0$ THEN

IF *Opinion Changed THEN $U_i > 0$*

ELSE $U_i < 0$

⇓

$$U_i\left(\sigma_i^{t+1}=\sigma_i^t,\sigma_i^t,\{\sigma_{j\neq i}\}\right)>U_i\left(\sigma_i^{t+1}=-\sigma_i^t,\sigma_i^t,\{\sigma_{j\neq i}\}\right)$$

⇓

individual chooses $\sigma_i^{t+1}=\sigma_i^t$ keeping its previous state

b) IF $I_i\left(\sigma_i^t,\{\sigma_{j\neq i}\}\right)>0$ THEN

$$U_i\left(\sigma_i^{t+1}=\sigma_i^t,\sigma_i^t,\{\sigma_{j\neq i}\}\right)<U_i\left(\sigma_i^{t+1}=-\sigma_i^t,\sigma_i^t,\{\sigma_{j\neq i}\}\right)$$

⇓

individual chooses $\sigma_i^{t+1}=-\sigma_i^t$ changing its state

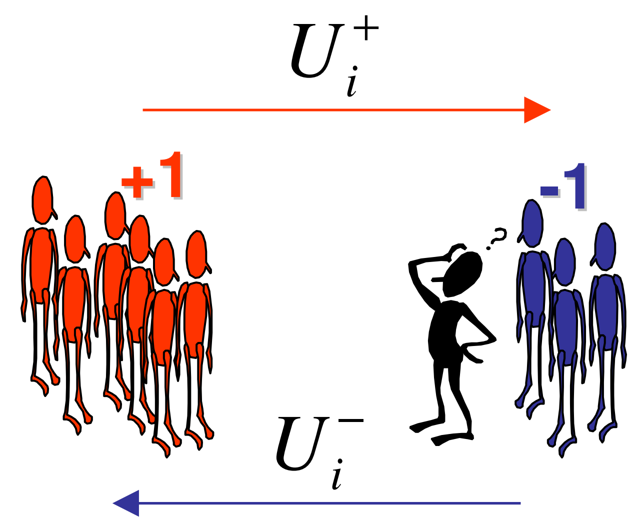
Generalized Model:

Generalized Utility Function

$$U_i = \frac{1}{2}\left[(1+\sigma_i^t)U_i^+(\sigma_i^{t+1},\sigma_i^t=+1,\mathbf{N_i},\varepsilon_i)+(1-\sigma_i^t)U_i^-(\sigma_i^{t+1},\sigma_i^t=+1,\mathbf{N_i},\varepsilon_i)\right]$$

$$U_i^{\pm} = \sigma_i' h_i^{\pm} \pm \sigma_i' b_i^{\pm} \pm \sigma_i' f^{\pm}(\mathbf{N_i}) + \varepsilon_i^{\pm}(\sigma_i')$$

Individual Preference	Self-Support (Agent's Inertia)	Social Influence: Support / Persuade	Random Factors



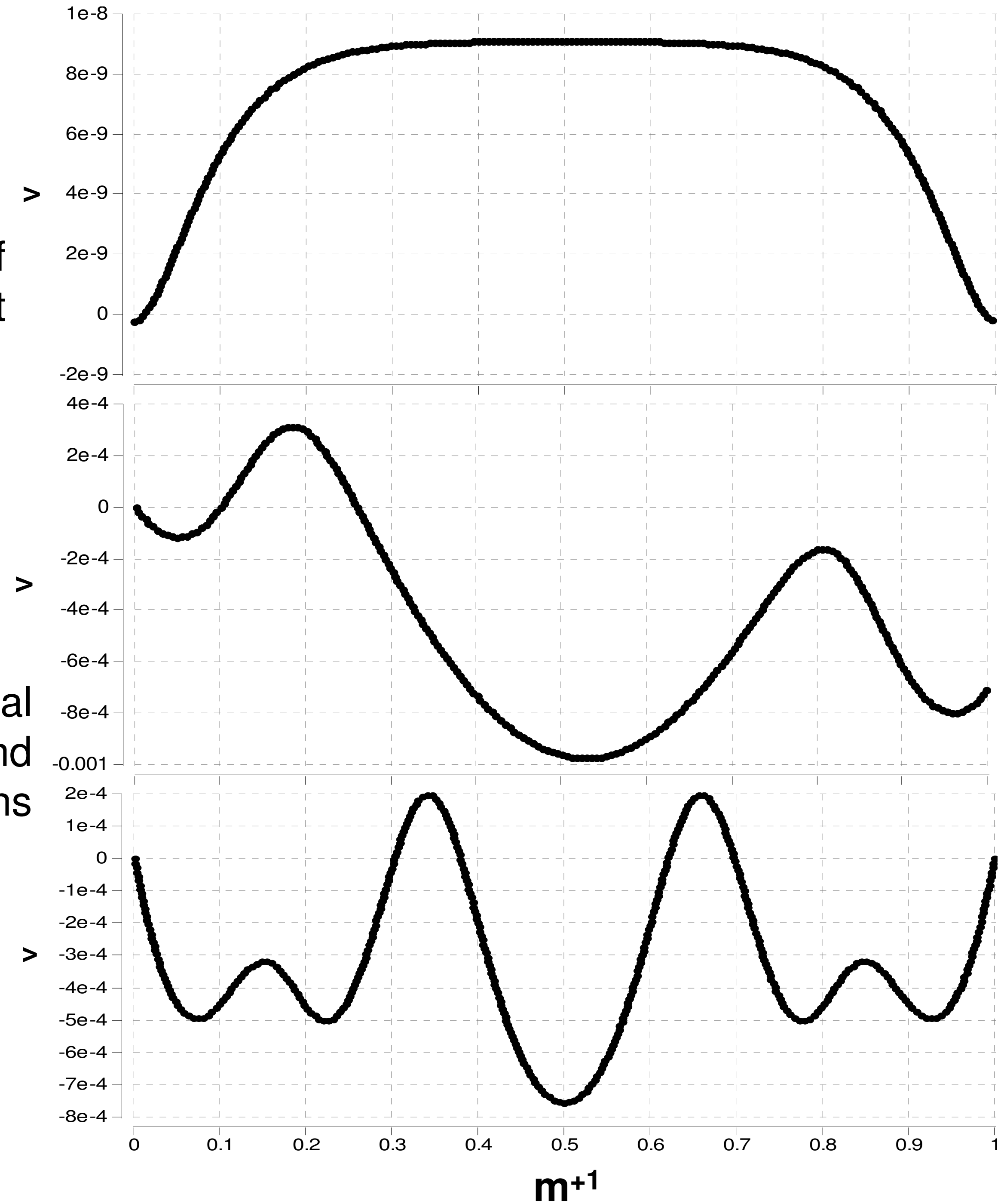
Generalized Model

Mean-Field Approximation

Potential for high values of self-support

Possible shapes of the potential for different supporting and persuading functions

Minima of the potential correspond to stationary states



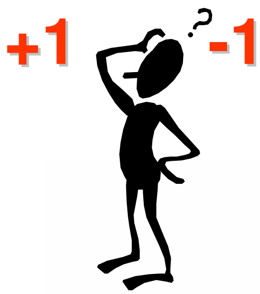
Binary Choice
for Heterogenous Agents

+1 **-1**

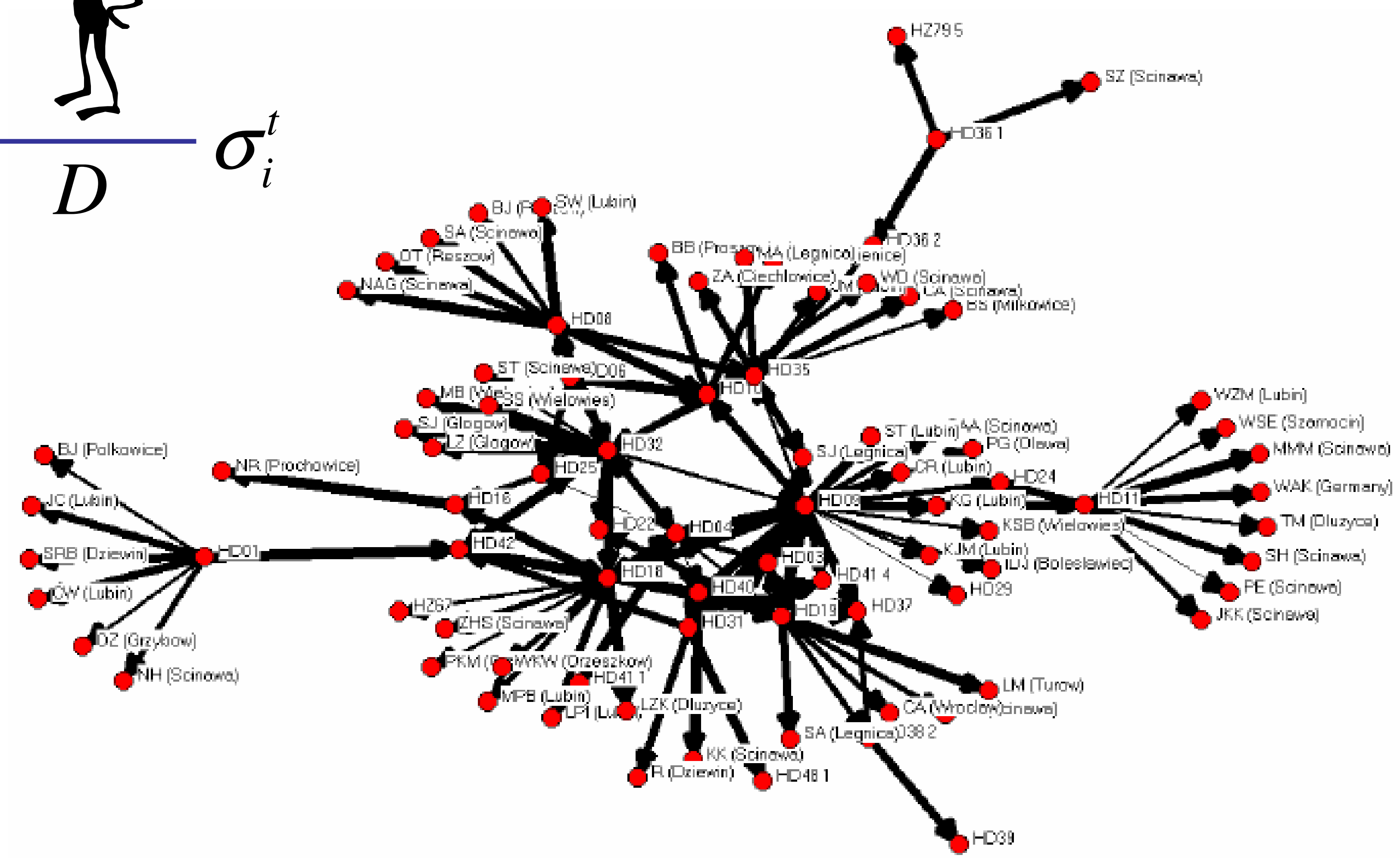
$$\sigma_i^{t+1} \xleftarrow{\quad D \quad} \sigma_i^{t}$$

Rigorous Aggregation of Agent-based Models into System Dynamics Models

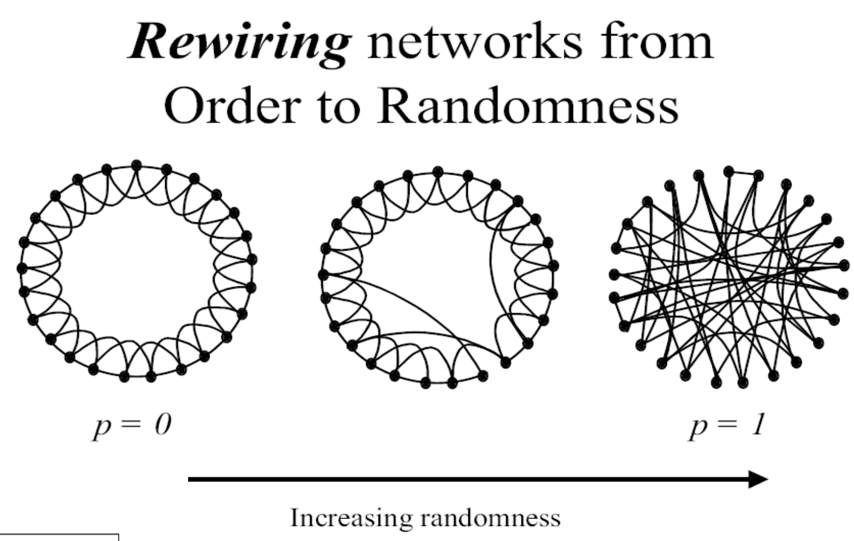
Binary Choice Models on Networks



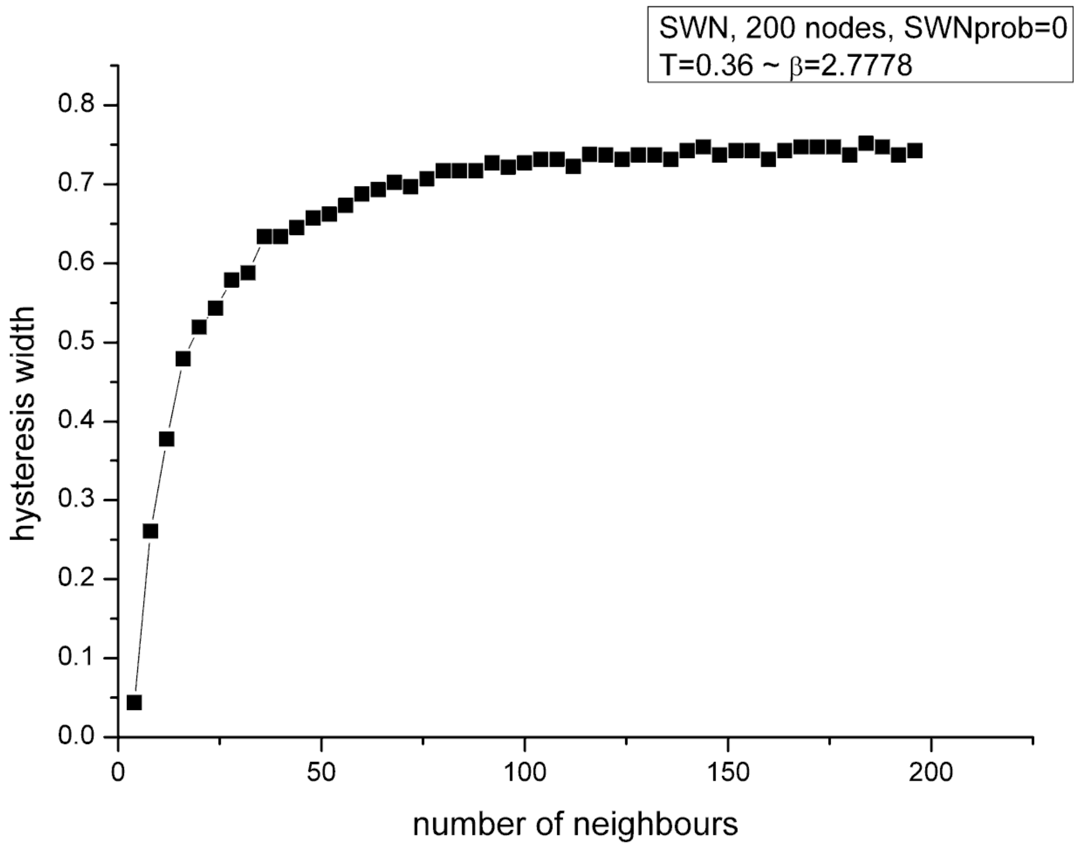
$$\sigma_i^{t+1} \xleftarrow{\;D\;} \sigma_i^{t}$$



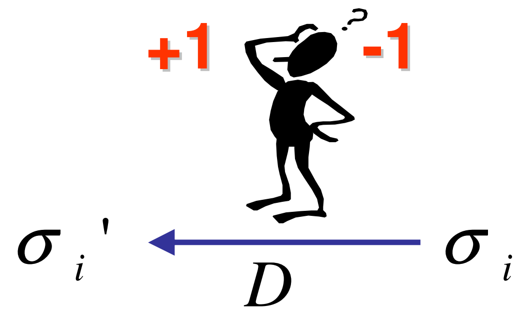
Brock - Durlauf Model
Small World Network



Dependence of hysteresis width on number of neighbors



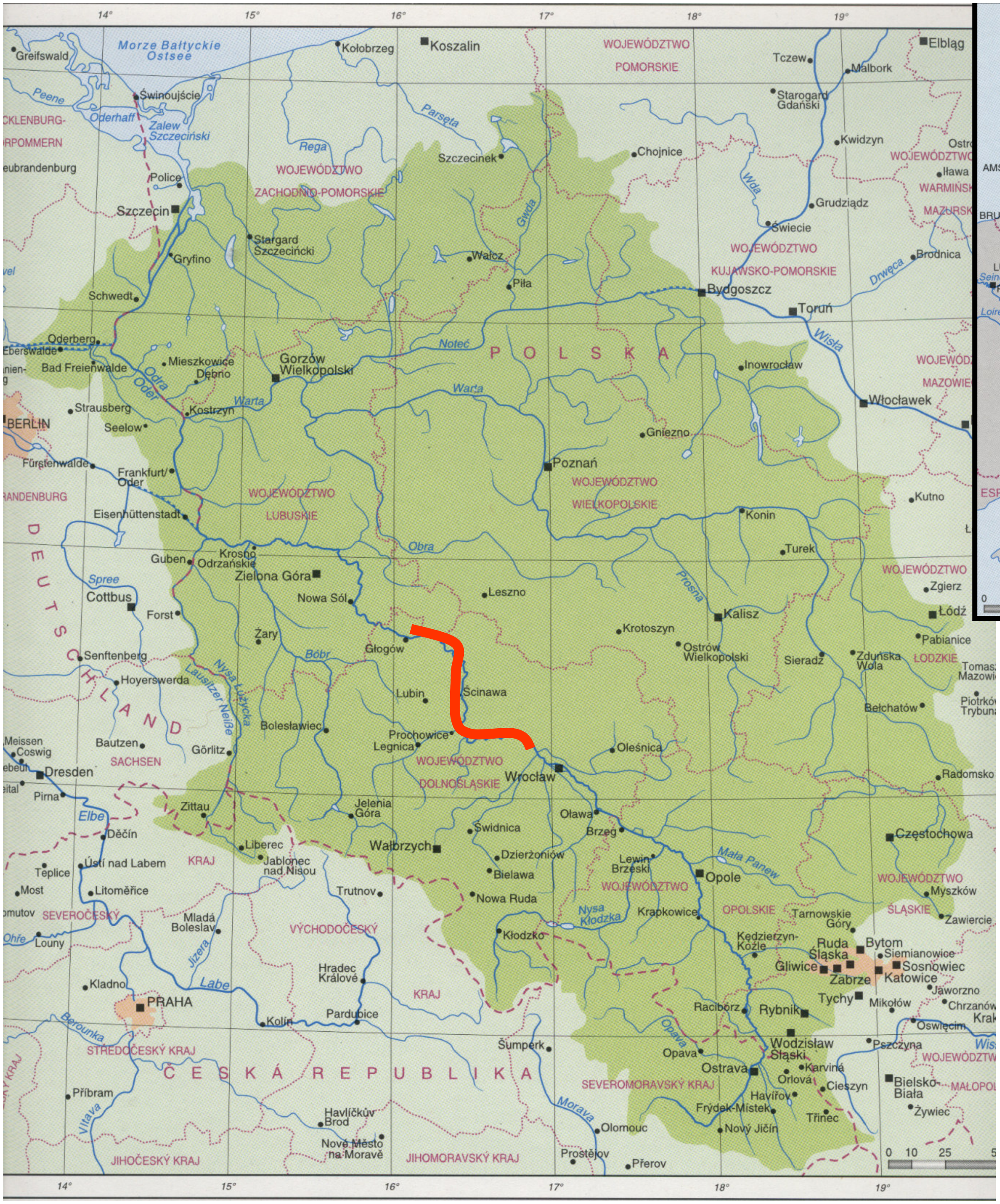
Summary



- Models of binary choice, opinion and attitude dynamics correspond with statistical mechanics spin models.
- Models formulated using utility function are mathematically equivalent with models using impact function.
- Self-support enables to introduce inertia in agents’ choices.
- Mean-field approximation (exact in complete-pairwise network) allows to find stationary states for wide class of models.
- Mean-field solutions provide reasonable approximation in network models for certain range of parameter values.

Maintenance of Drainage System in the Odra River Valley

Validation of Agent-Based Model through Role-Playing Simulation



Odra River Watershed

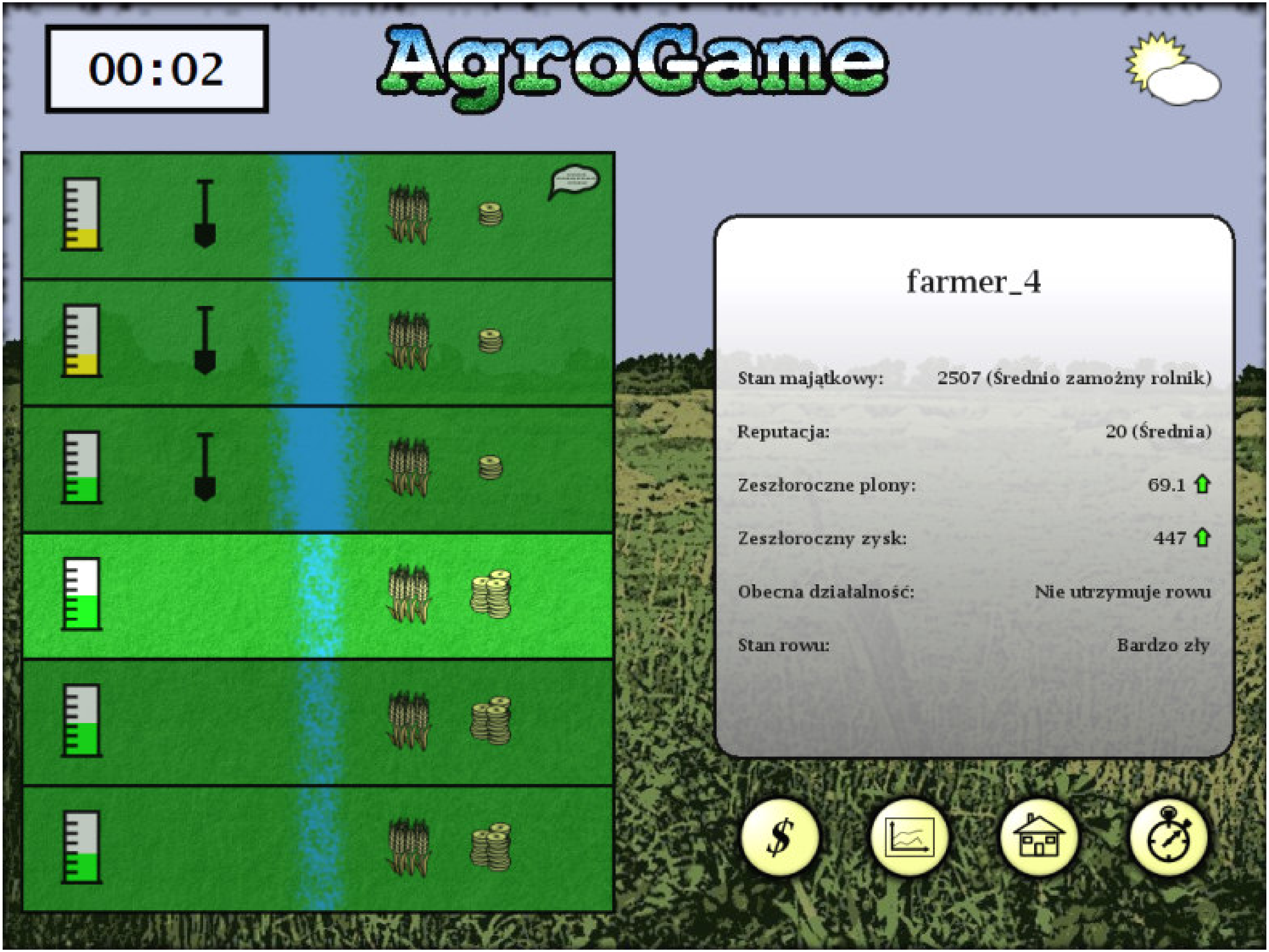
Problem:

Land Amelioration System is not maintained properly due to institutional changes.

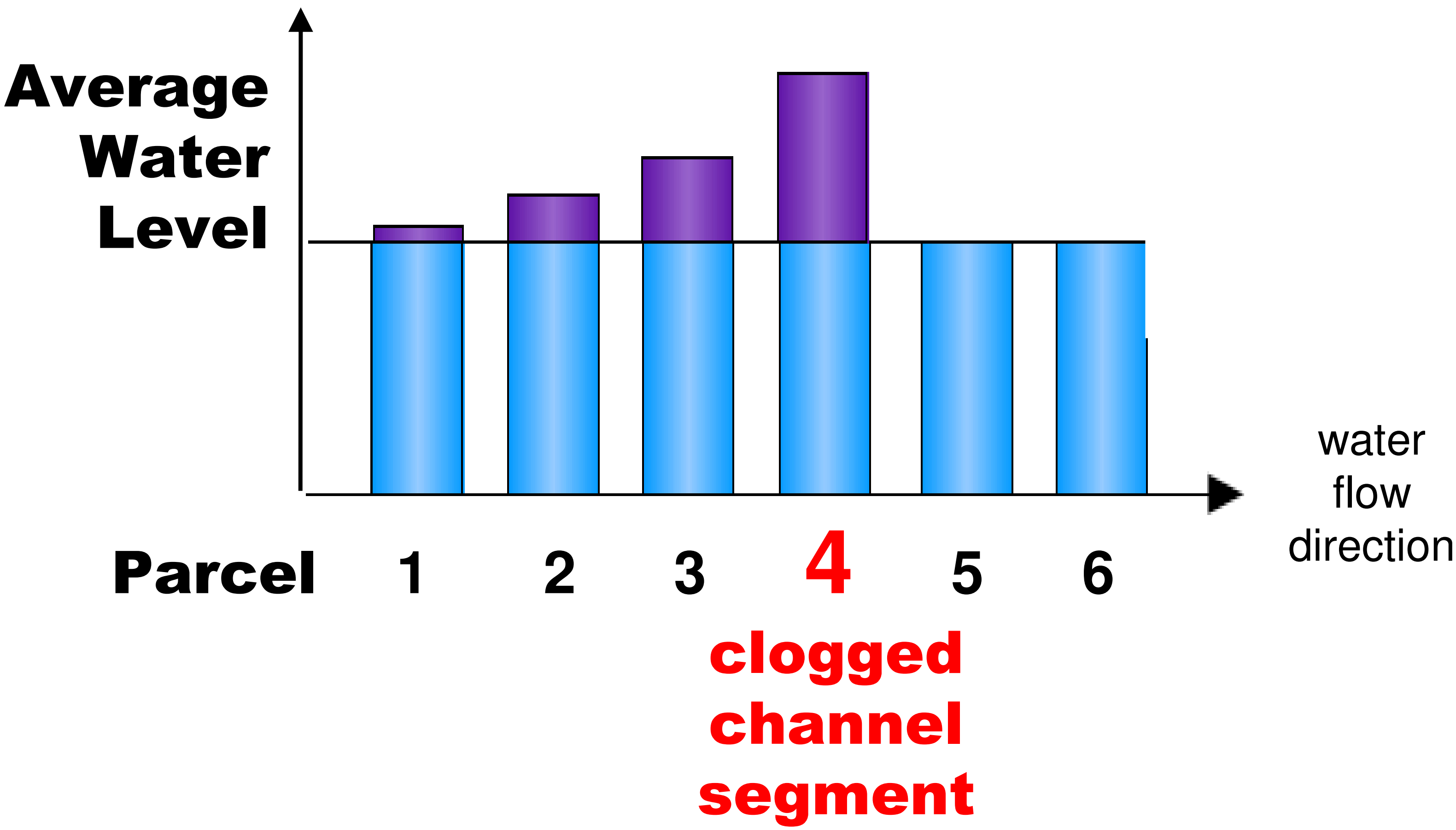
Maintenance of Land Amelioration System in Odra River Valley

Data and Information Sources:

- Expert Judgment
- Workshops with Farmers
- Social Field Research
  (semistructured interviews with farmers in the study area)
- Role-Playing Game


00:02
AgroGame
farmer_4
Stan majątkowy: 2507 (Średnio zamożny rolnik)
Reputacja: 20 (Średnia)
Zeszłoroczne plony: 69.1
Zeszłoroczny zysk: 447
Obecna działalność: Nie utrzymuje rowu
Stan rowu: Bardzo zły
$

Spatial average water level distribution



Game results – qualitative – decision rules

Economic

Maintain:	- Reduce Waterlogging which Increases Yields and therefore Profits
	- Increases Profits of other Farmers
Not Maintain:	- Unnecessary cost

Social

Maintain:	- Reciprocity – „we will all be better of”
	- Interconnectedness – „it is one system - all of us should maintain”
	- Good relations with neighbours
Not Maintain:	- ”Others do not maintain so do I”
	- ”It should be done by the State”

Technical

Maintain:	- If wet year then maintain otherwise not
Not Maintain:	- Lack of technical abilities

Game results – quantitative
Game 2

Significant correlations:

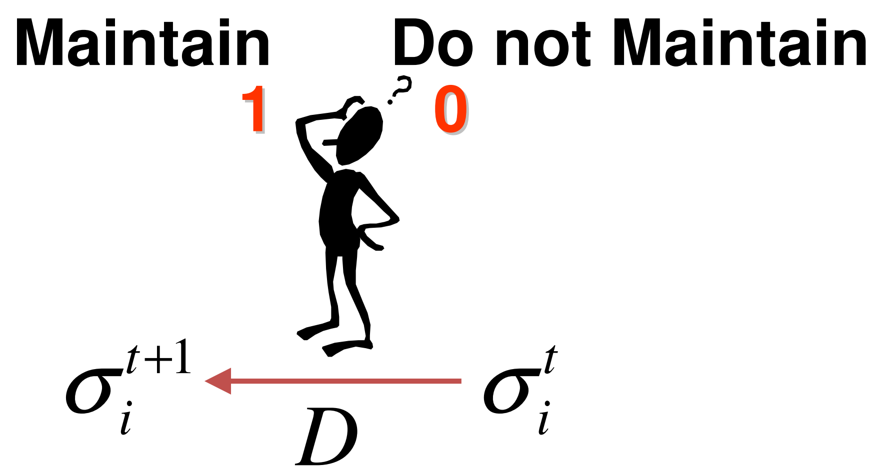
- **Previous choice**
- **Critisism or Praise**
- Profit
- **Neighbours' Choice**
- Others' Choice
- Weather

Used statistics:

- Goodman-Kruskal
- Chi2
- Fisher

Best logistic regression model:

$$\Pr(\sigma_i^{t+1} = 1 \mid \sigma_i^t) = \frac{1}{1 + \exp(-(d_0 + d_1 * \sigma_i^t))}$$



Game results – quantitative
Game 3

Significant correlations:

- Previous choice
- **Critisism or Praise**
- Profit
- **Neighbours' Choice**
- **Others' Choice**
- Weather

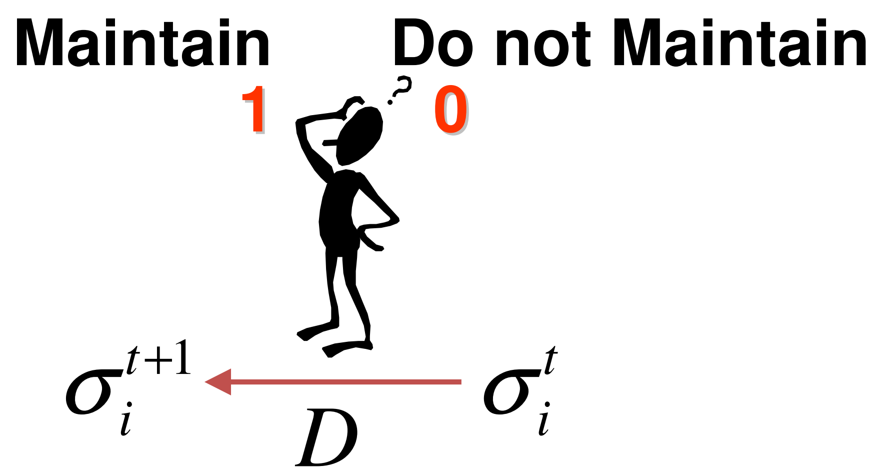
Used statistics:

- Goodman-Kruskal
- Chi2
- Fisher

Best logistic regression model:

$$\Pr(\sigma_i^{t+1} = 1 \mid I) = \frac{1}{1 + \exp(-(d_0 + d_1 I))}$$

I – Sum of Others' Choices

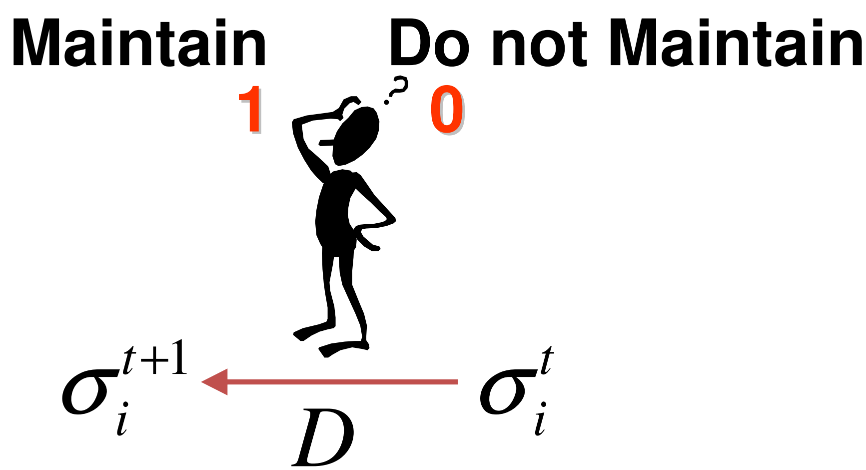


Modeling Agents' Decisions
Economic Factors

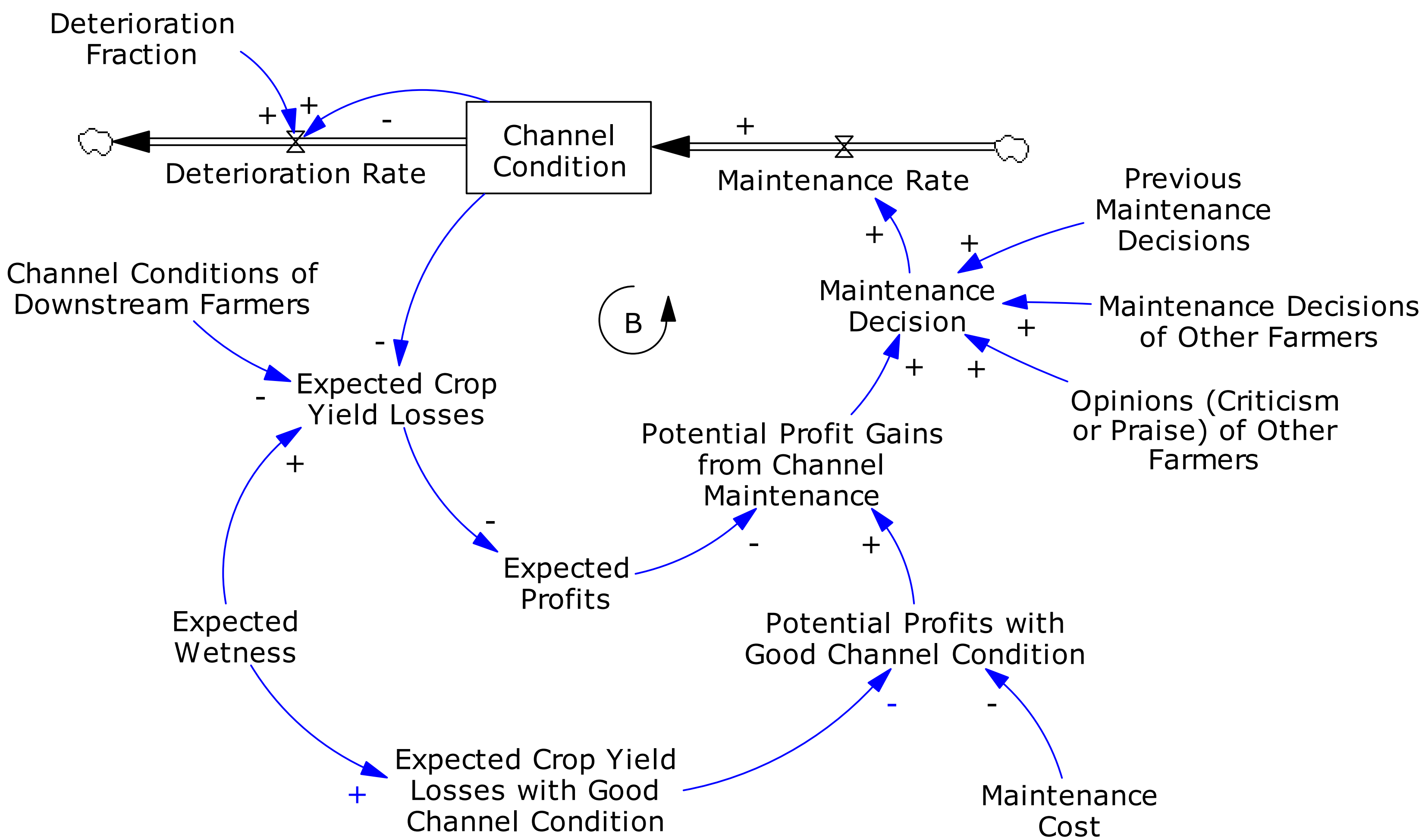
Adding economic factor *G* which influence Agent's choice X

$$\Pr(\sigma_i^{t+1} = 1 \mid I) = \frac{1}{1 + \exp(-(d_0 + d_1 I + d_2 G))}$$

G – Profit gains from channel maintenance



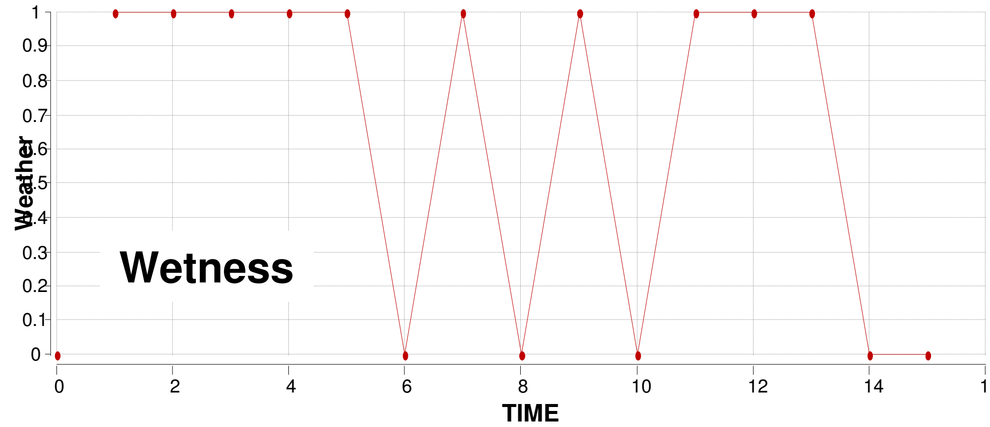
Combined Social-Ecological Model

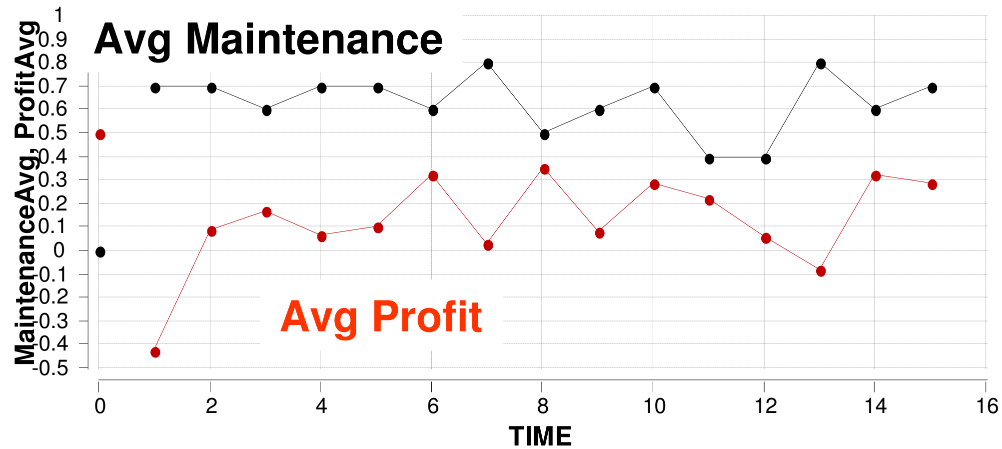


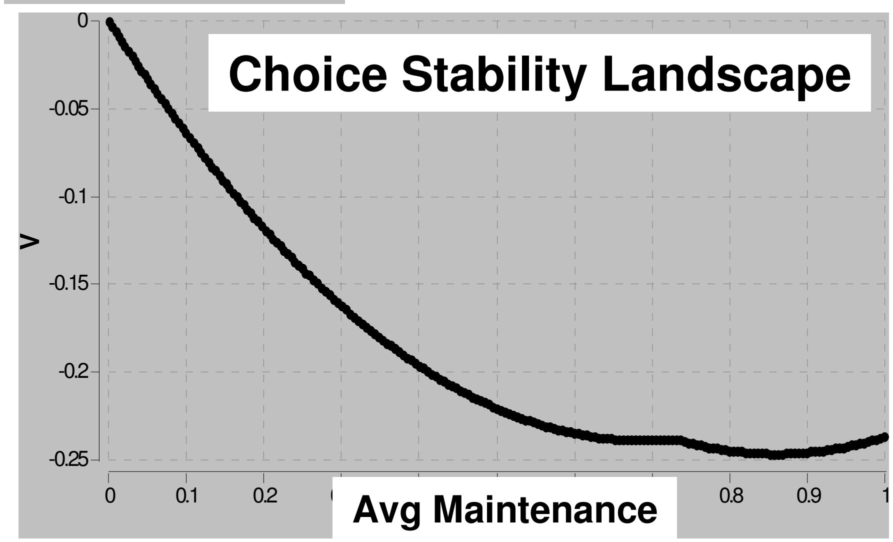
Base Model Run

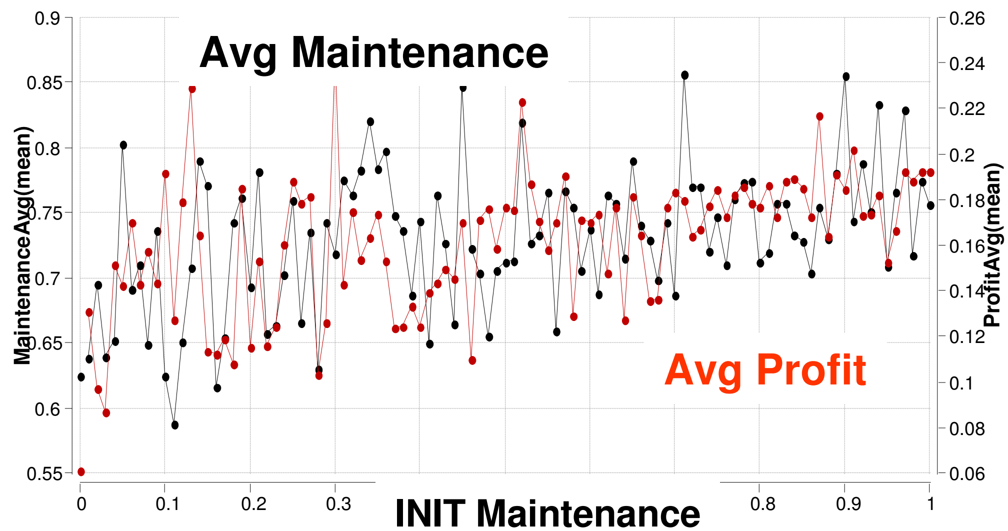
Decision paramters estimated from experimental data

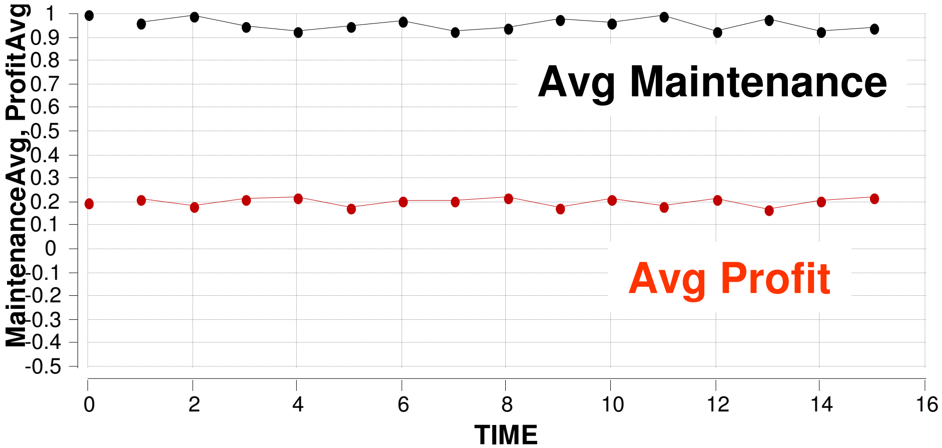
d0 =	0.8
d1 =	1
d2 =	0
SocialThreshold =	0.73
Production_Cost =	0.5
Maintenance_Cost =	0.3
Wetness =	0.66
Losses0 =	0.5

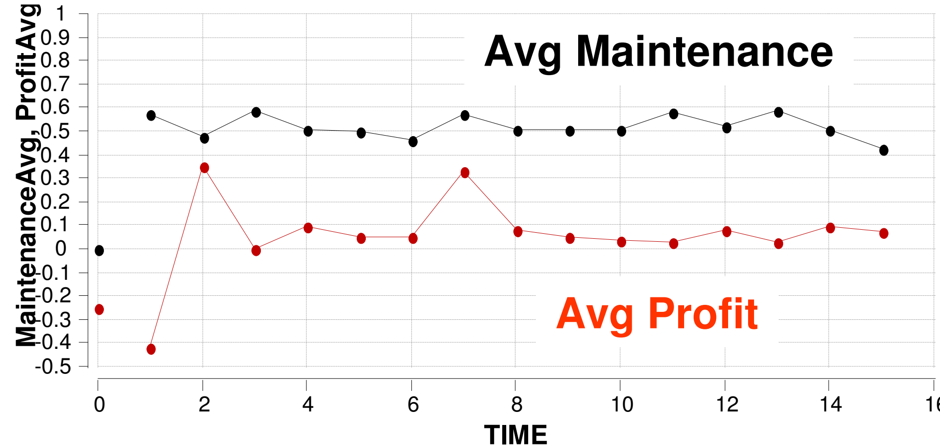








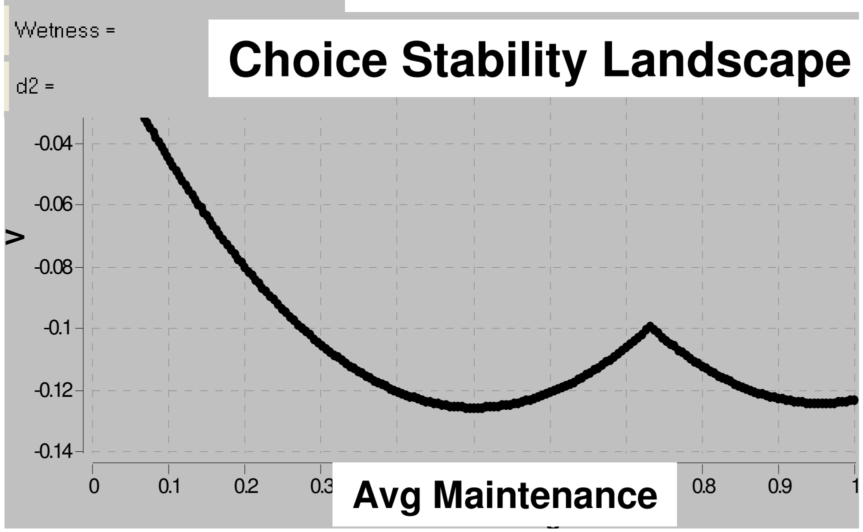


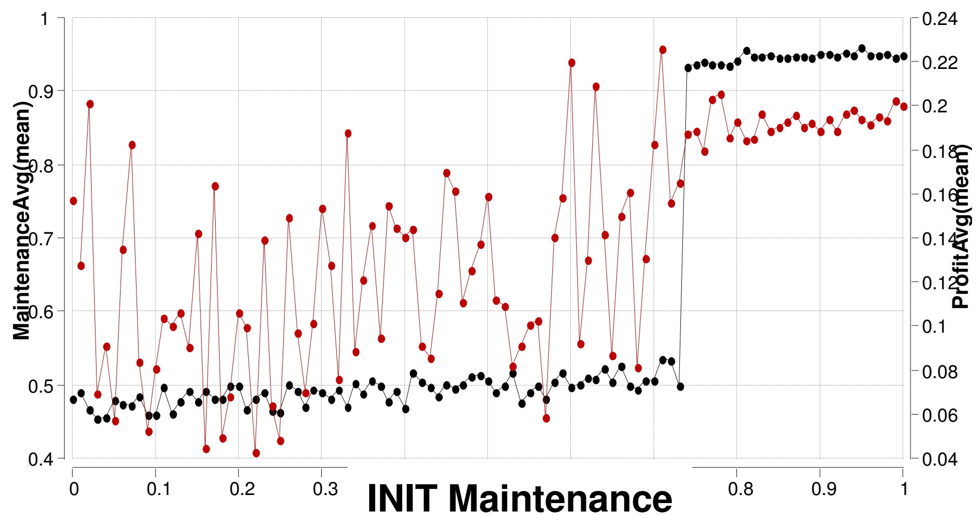


Model Run

Parameters:
High social influence

r =	1
Production_Cost =	0.5
Maintenance_Cost =	0.3
Losses0 =	0.5
d0 =	0
d1 =	3
SocialThreshold =	0.73
INIT_Maintenance =	0
Wetness =	
d2 =	



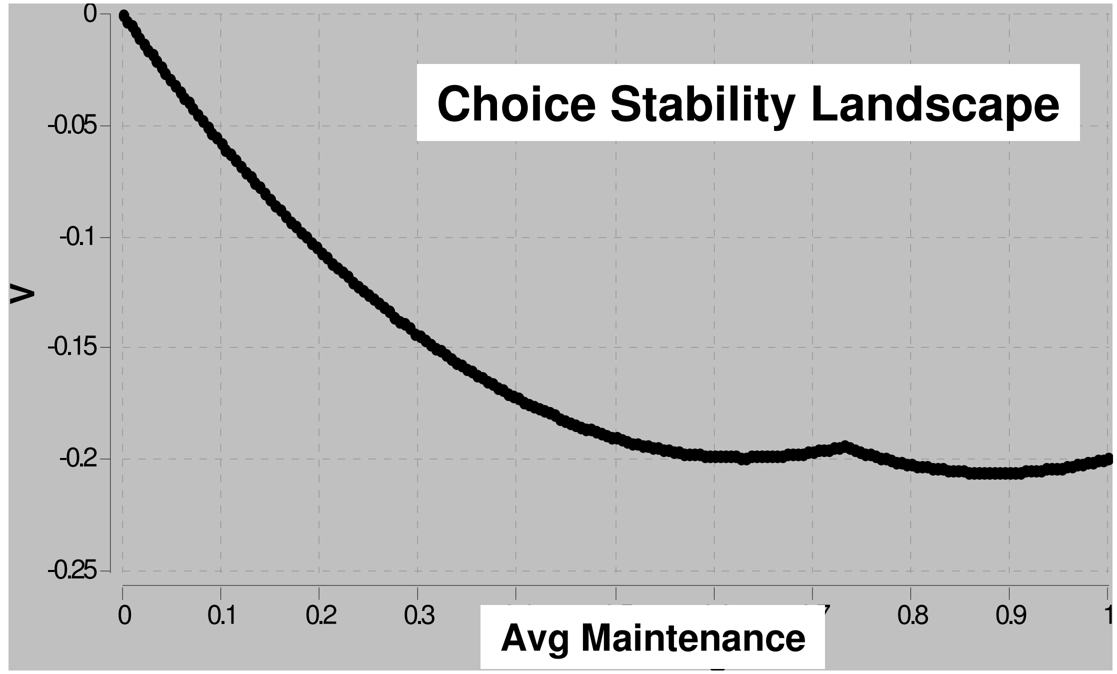


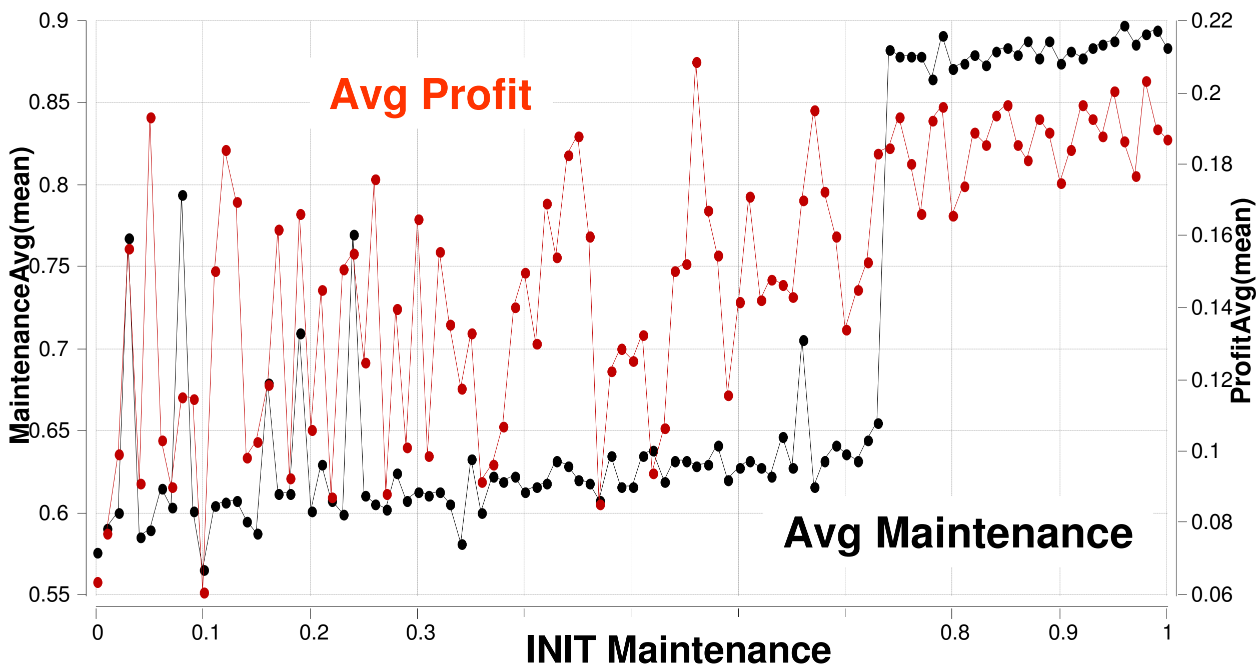
Model Run

Parameters:

Include economic motivation – maintenace more profitable

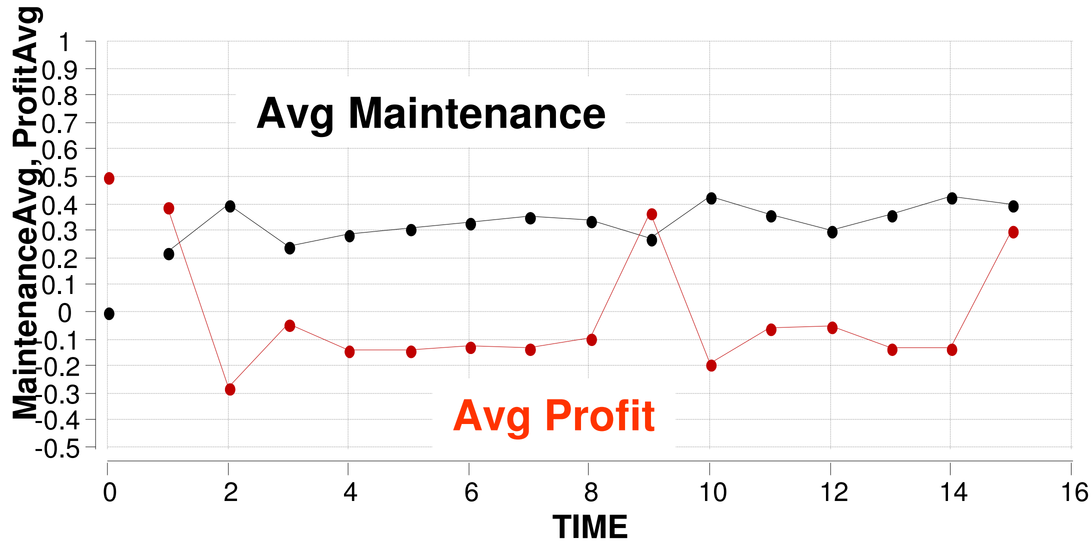
r =	1
Production_Cost =	0.5
Maintenance_Cost =	0.3
Losses0 =	0.5
d0 =	0.5
d1 =	1.5
SocialThreshold =	0.73
INIT_Maintenance =	1
Wetness =	0.7
d2 =	1



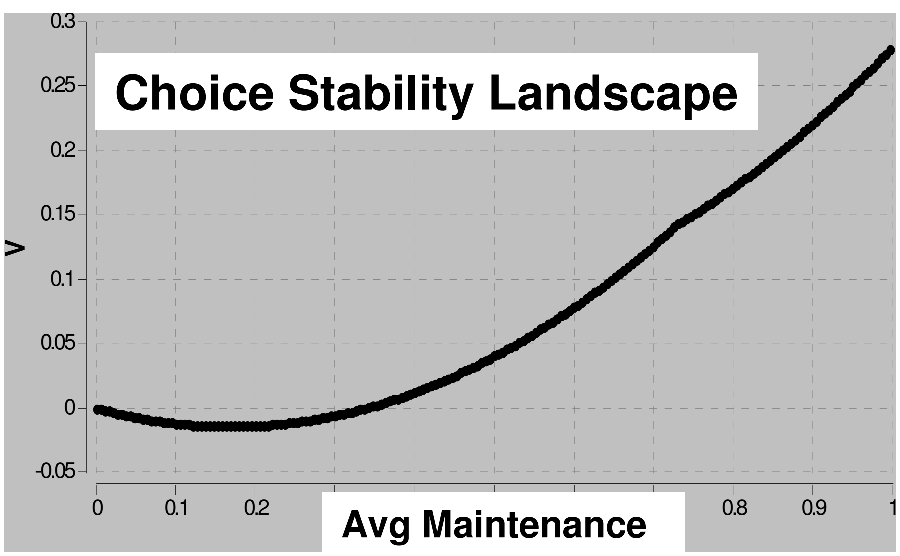


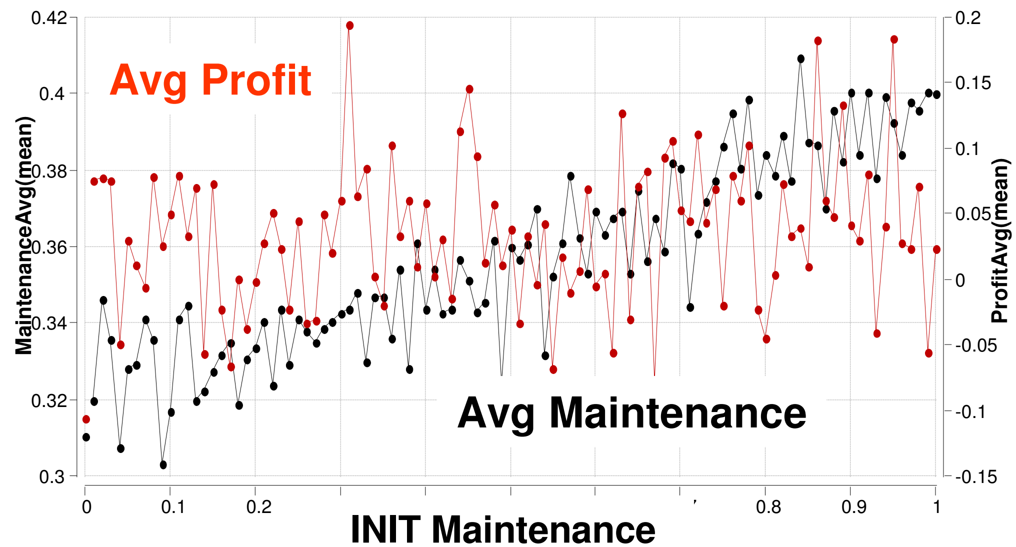
d0 =	0.1
d1 =	1
d2 =	10
SocialThreshold =	0.73
Production_Cost =	0.5
Maintenance_Cost =	0.5
Wetness =	0.66
Losses0 =	0.5

Model Run



Parameters:
Include economic motivation – maintenace less profitable





$$\sigma_i^{t+1} \xleftarrow[D]{} \sigma_i^{t}$$

Social Economics Models

Example: Brock-Durlauf Model – Mean-Field Approximation

$$\Pr\left(\sigma_i = \sigma\right) = \Pr\left(U_i(\sigma) > U_i(-\sigma)\right) = \Pr\left(\varepsilon_i(-\sigma) - \varepsilon_i(\sigma) < 2\sigma z_i\right)$$

$$z_i = h_i + \sum_{i \neq j} J_{ij}\sigma_j$$

$$\Pr(\sigma_i = \sigma) = \begin{cases} F_{\beta_i}^{\log}(2z_i) \text{ for } \sigma = +1 \\ 1 - F_{\beta_i}^{\log}(2z_i) \text{ for } \sigma = -1 \end{cases}$$

$$E\left(\sigma_i\right) = \Pr(\sigma_i = +1) - \Pr(\sigma_i = -1) = 2F_{\beta_i}^{\log}(2z_i) - 1$$

In a special case

deterministic private incentives are identical accross individuals $h_i = h$

distributions of random terms are identical accross individuals $\beta_i = \beta$

each person cares only about the average choice of others $J_{ij} = \dfrac{J}{I-1} \quad \forall j \neq i$

expected average choice in a population is

$$m = \tanh\left(\beta h + \beta J m\right)$$

This is mean field (Currie-Weiss) approximation for ferromagnetism.

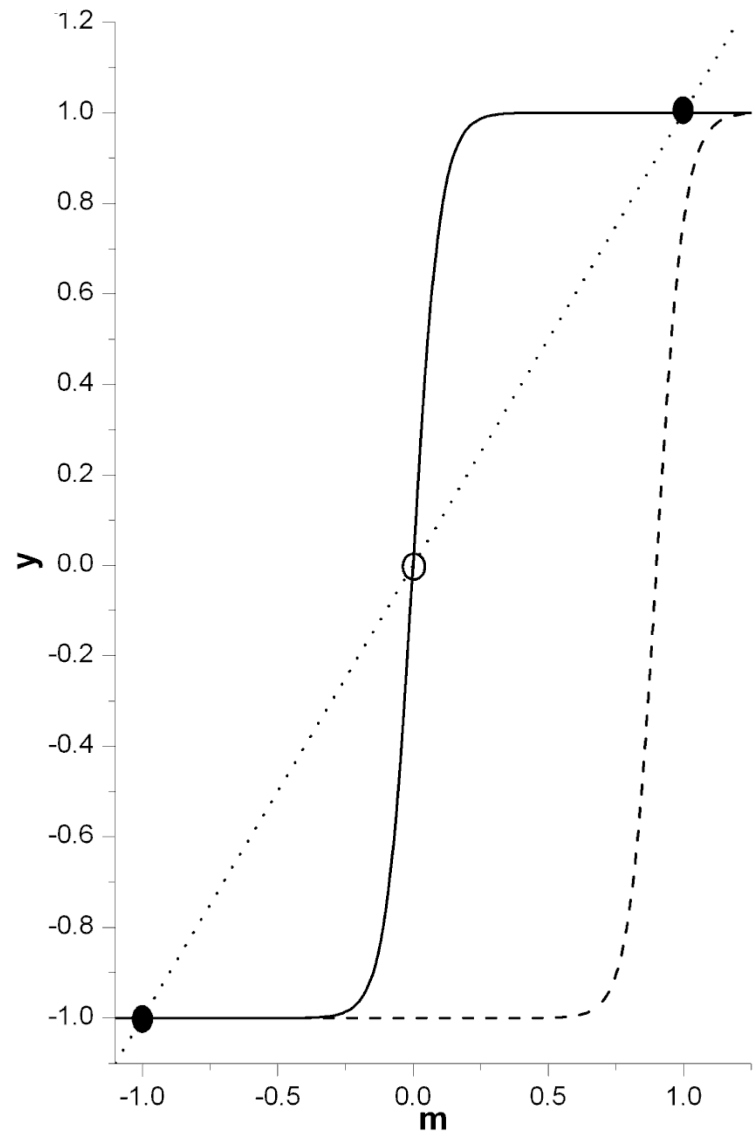
Social Economics Models

Example: Brock-Durlauf Model – Mean-Field Approximation

Stationary States

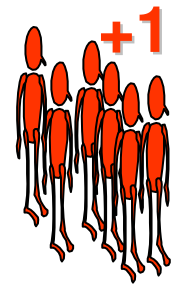
Full circles corresponds to stable stationary states and open circles to unstable stationary states.

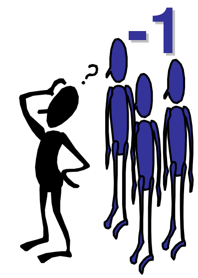
$$m = \tanh\left(\beta h + \beta J m\right)$$



Binary Choice Models

Mean Field Approximation – Stationary States - Potential



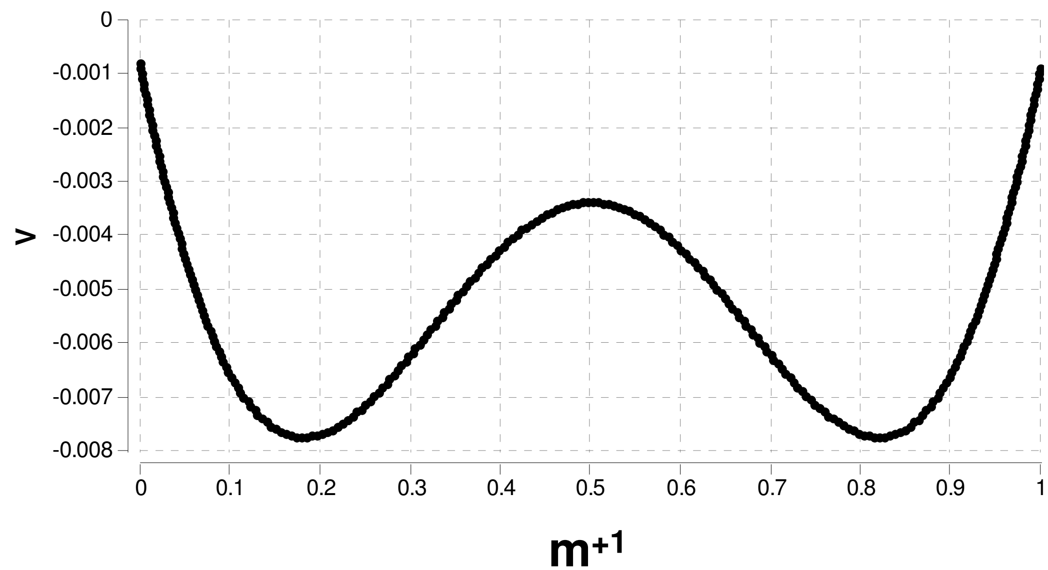


$$\frac{\mathrm{d}m^{+1}}{\mathrm{d}t} = \Pr(\sigma_i' = +1 \mid \sigma_i = -1)m^{-1} - \Pr(\sigma_i' = -1 \mid \sigma_i = +1)m^{+1}$$

$$m^{+1} + m^{-1} = 1; \quad m^{+1} - m^{-1} = m$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = H(m)$$

$$H(m) = -\frac{\mathrm{d}V}{\mathrm{d}x}$$



Minima of the potential correspond with stable fixed points, maxima – unstable fixed points

Dynamic Social Psychology Models

Holyst - Kacperski Model

Decision Rule:

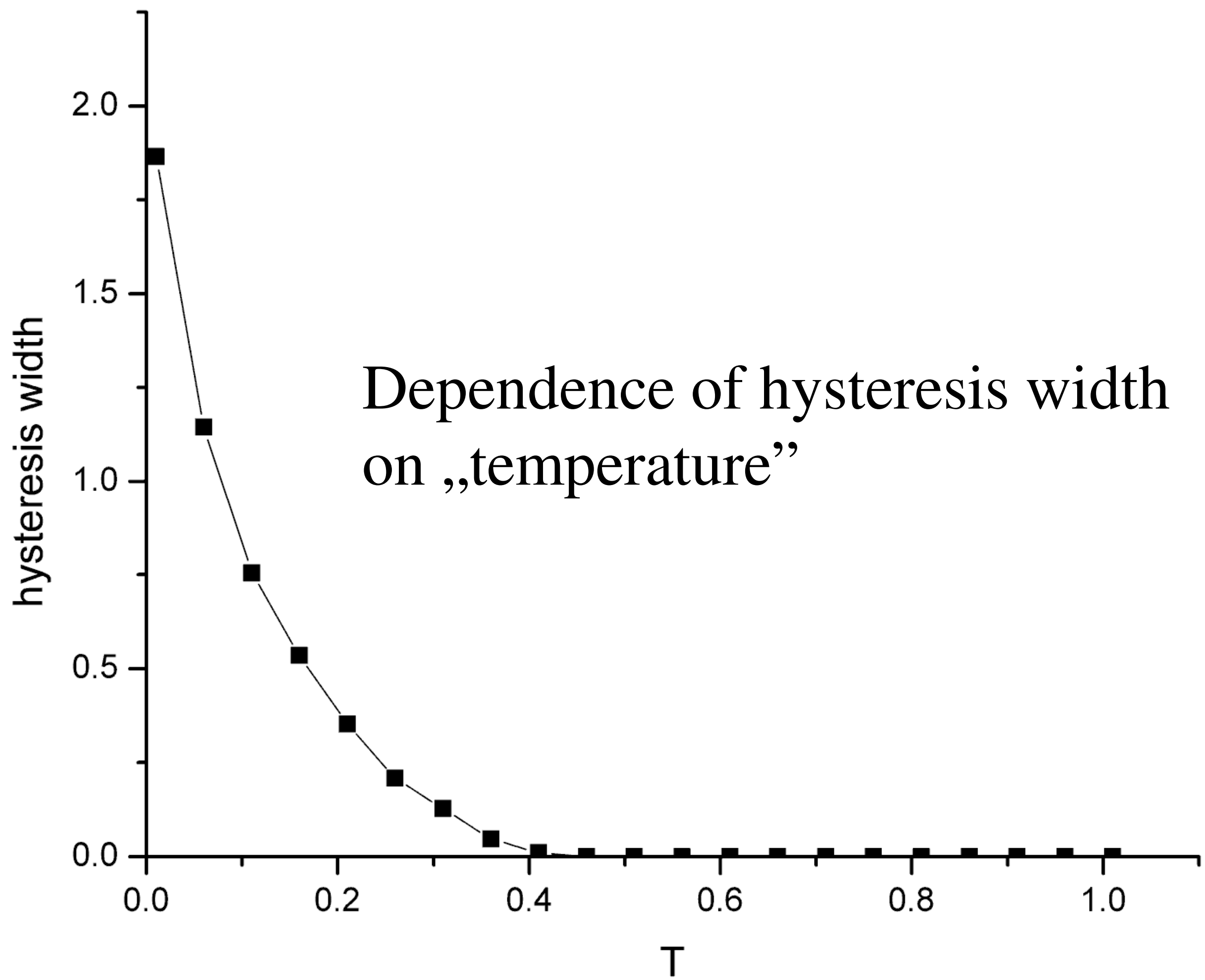
$$\sigma_i' = \begin{cases} \sigma_i \text{ with probability } \dfrac{1}{1+\exp(\beta I_i)} \\ -\sigma_i \text{ with probability } \dfrac{1}{1+\exp(-\beta I_i)} \end{cases}$$

Impact Function

$$I_i = -s_i b - \sigma_i h - \sum_{j \neq i} \frac{s_j \sigma_i \sigma_j}{g\left(d_{ij}\right)}$$

s_i - strength of influence
b - self-support
h - additional (external) influence which may be regarded as a global preference towards one of the opinions stimulated by mass–media, government policy, etc.
1/β - may be interpreted as a "social temperature" describing a degree of randomness in the behaviour of individuals,

Brock - Durlauf Model Results:



Mean-Field Approximation of the Generalized Model

$$P(\sigma_i' = -1 \mid \sigma_i = -1) = P\left(U_i^-(\sigma_i' = +1) - U_i^-(\sigma_i' = -1) > 0\right) = P\left(m_i^{Th-} < m\right) \equiv F^{Th-}(m)$$

$$P(\sigma_i' = -1 \mid \sigma_i = +1) = P\left(U_i^+(\sigma_i' = -1) - U_i^+(\sigma_i' = +1) > 0\right) = P\left(m_i^{Th+} > m\right) \equiv 1 - F^{Th+}(m)$$

$$m = \frac{F^{Th+}(m) + F^{Th-}(m) - 1}{1 + F^{Th-}(m) - F^{Th+}(m)}$$

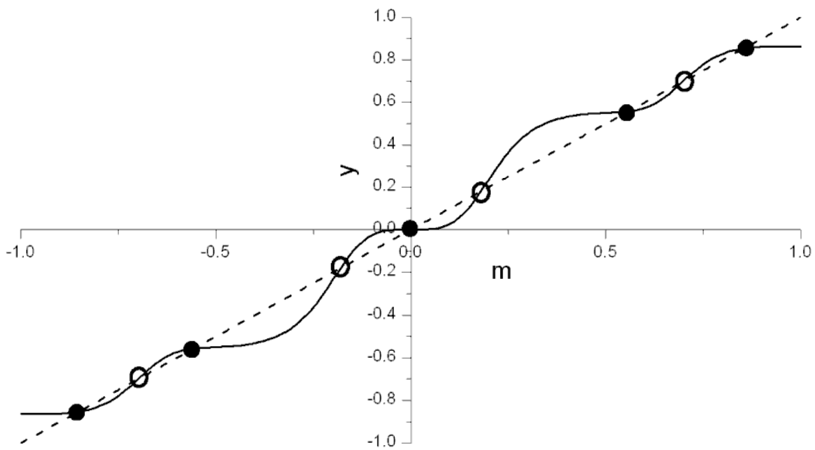
$$F^{Th-}(m) = 1 - F^h\left(b^- + f^-(m)\right)$$

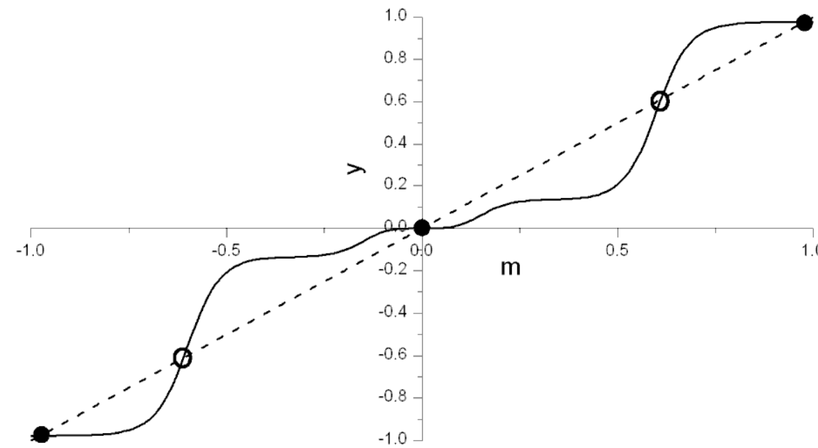
$$F^{Th+}(m) = 1 - F^h\left(-b^+ - f^+(m)\right)$$

$F^h(x)$ - cumulative distribution function of *h*

Mean-Field Approximation of the Generalized Model – Stationary States

Graphical analysis of the generalized model for the selected parameter values





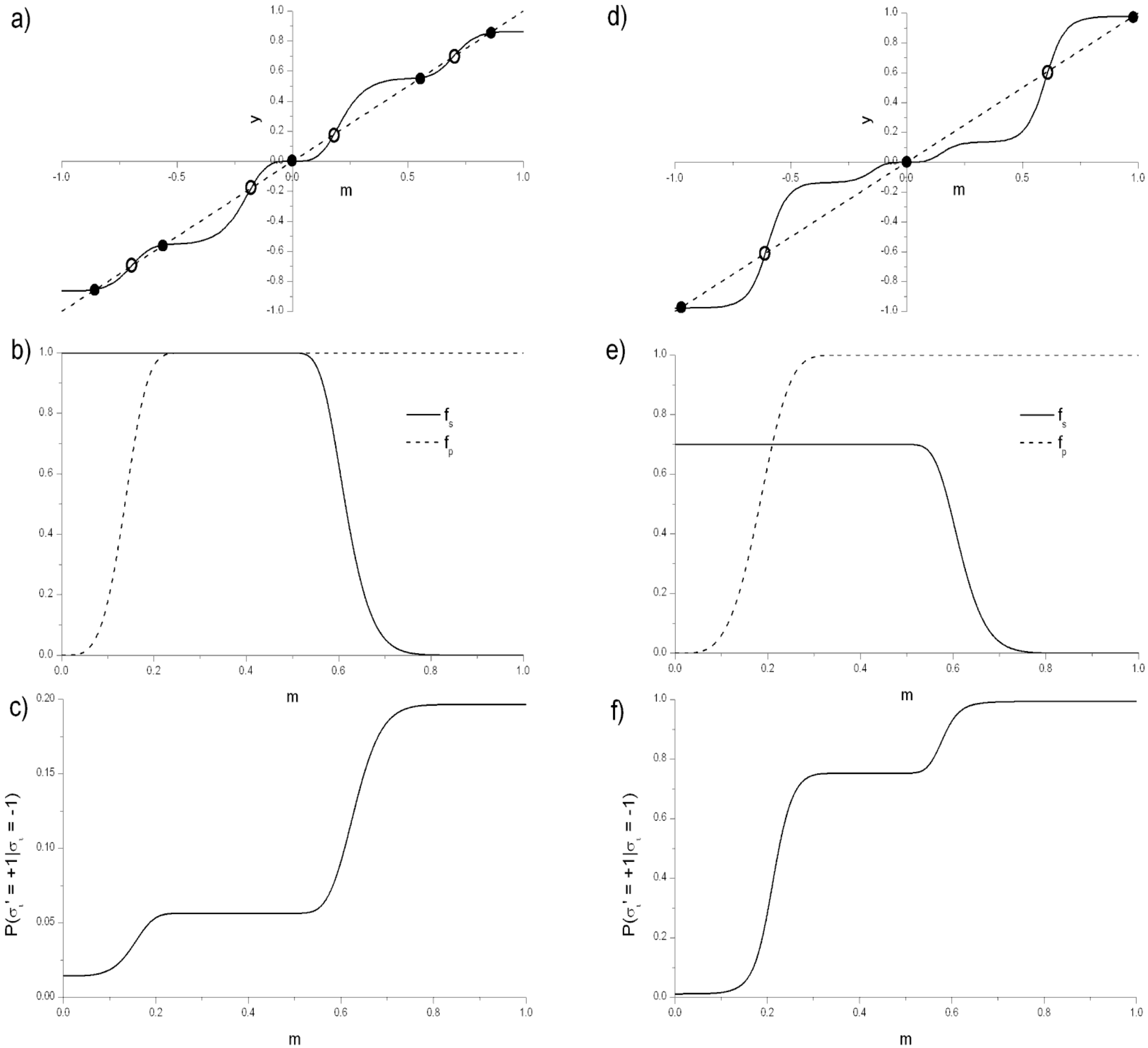
Plot of the left (dashed line) and right-hand (solid line) side of:

$$m = \frac{F^{Th+}(m) + F^{Th-}(m) - 1}{1 + F^{Th-}(m) - F^{Th+}(m)}$$

● - stable stationary states,

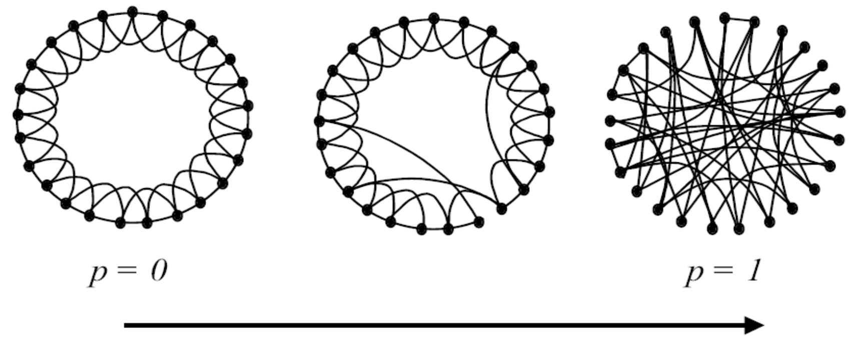
○ - unstable stationary states.

Mean-Field Approximation of the Generalized Model

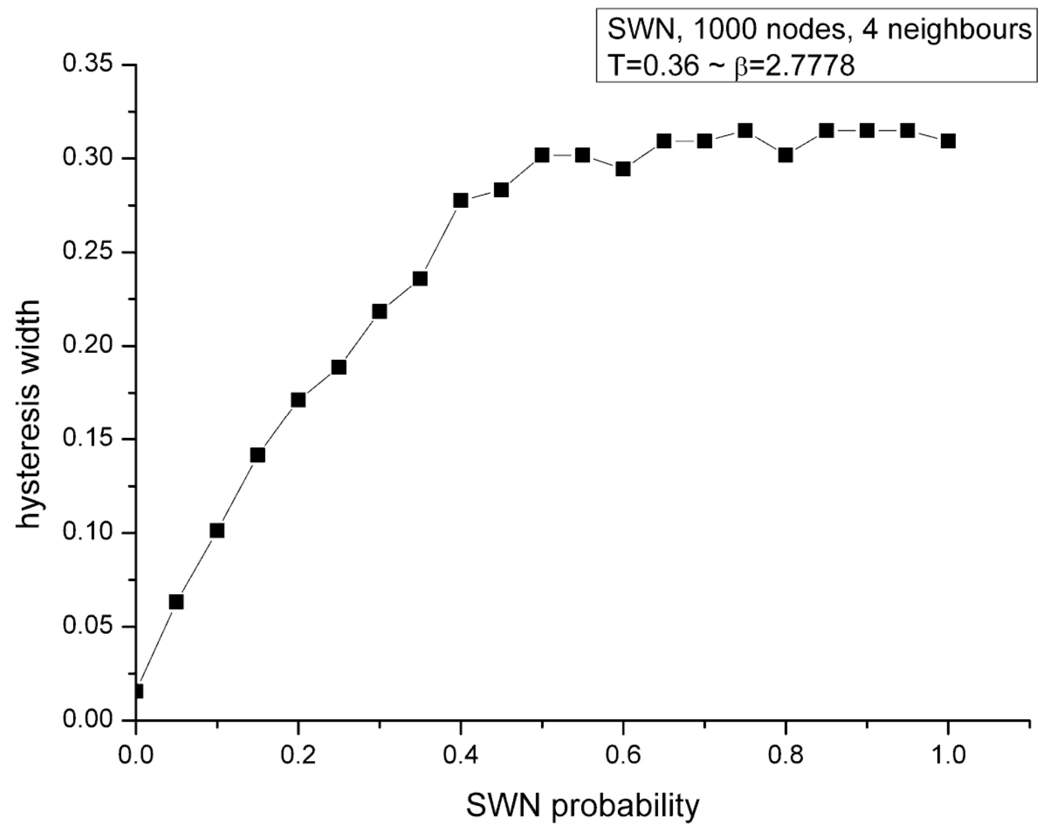


Brock - Durlauf Model
Small World Network

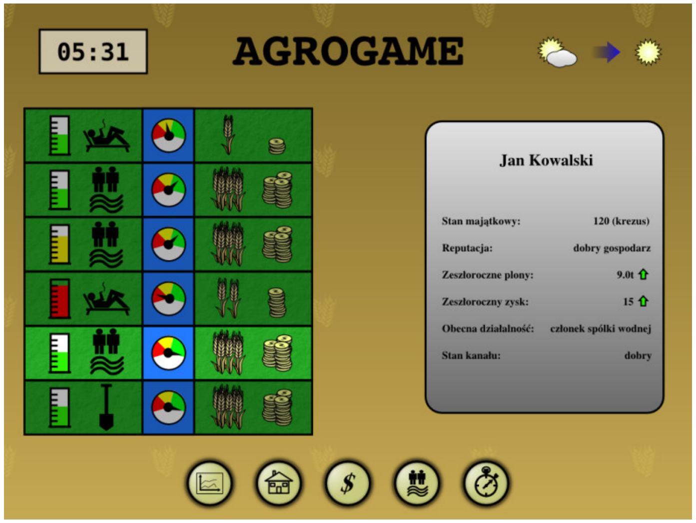
Rewiring networks from
Order to Randomness

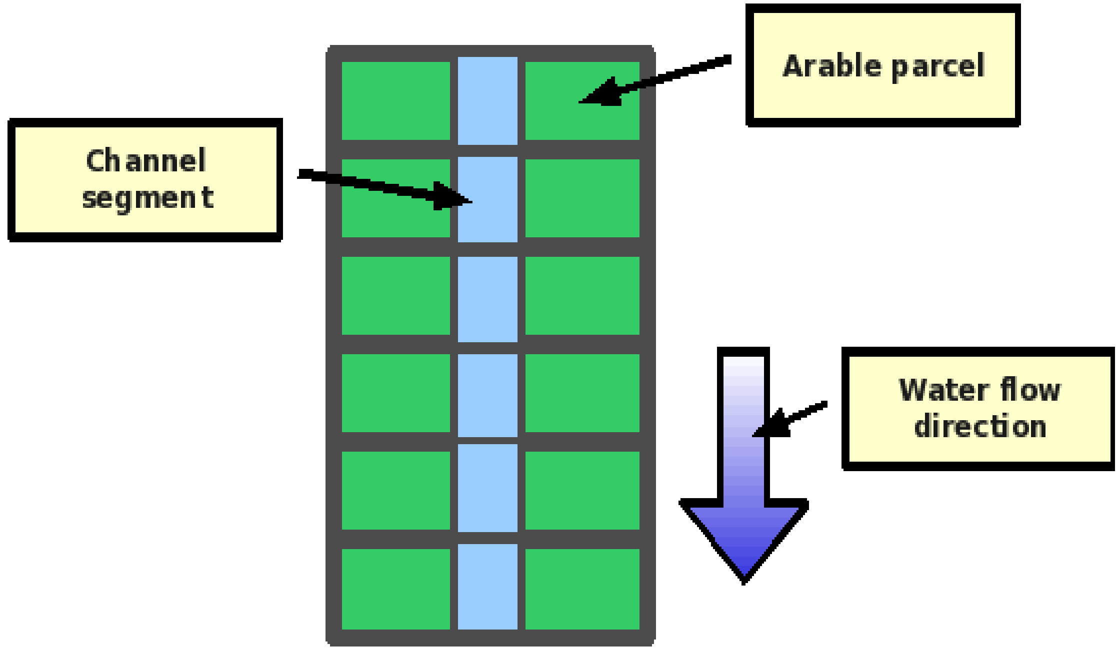


Dependence of hysteresis width
on rewiring probability



Agrogame





Basic model assumptions:

- The model's world is quasi two-dimensional.
- Parcels are homogeneous in terms of area and hydrological properties. They may differ in terms of channel segment condition.
- The time step is one year.
- The terrain under the parcels has a small, homogeneous slope along the channel's axis.
- Weather conditions are the same for all parcels.
- Weather conditions in one year do not influence the amount of water in the system in the next year.

Modeling Agents' Decisions
Social Influence

Agent choice X is influenced by its social environment.
Social influence *I* is defined as a random variables taking values 0 or 1 depending on the sum of others' choices.

I=0	1,2 or 3	other agents choosing 1
I=1	4 or 5	other agents choosing 1

$$\Pr(X' = 1 \mid I) = \frac{1}{1 + \exp(-(d_0 + d_1 I))}$$

d_0	0.8397
d_1	1.0412