Mathematics and the Social Sciences: The Challenge Ahead

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Abstract

The current financial crisis confronts decision makers, scholars, and the public at large with the fact that the prevailing understanding of markets leaves room for considerable improvement. Such improvement may actually be a matter of great practical urgency. Nevertheless, it will require patient research combined with pragmatic action. It is time to reconsider the metaphor of an equilibrium between supply and demand that lies at the root of much current social science work, ranging well beyond economics. A more fundamental metaphor seems to be the one of solving coordination problems via conventions. It leads to challenging questions about the dynamics of social networks, helps to improve our understanding of the interactions between financial markets and the so-called real economy, and provides the starting point for a promising research program at the interface of mathematics and the social sciences.

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1 Financial Crises and Mathematical Metaphors

In October 2008 Dani Rodrik - professor of political economy at Harvard University and recipient of the Social Science Research Council's Hirschman Prize - made the following comment on the current financial crisis: "what will the post-mortem on Wall Street show? That it was a case of suicide? Murder? Accidental death? Or was it a rare instance of generalized organ failure? We will likely never know.

The regulations and precautions that lawmakers will enact to prevent its recurrence will therefore necessarily remain blunt and of uncertain effectiveness.

That is why you can be sure that we will have another major financial crisis sometime in the future, once this one has disappeared into the recesses of our memory. You can bet your life savings on it. In fact, you probably will" (Rodrik 2008).

Two months later, George Soros – financial speculator of proven success and committed critic of today's financial institutions – made the following comment on the same crisis: "The salient feature of the current financial crisis is that it was not caused by some external shock like OPEC raising the price of oil or a particular country or financial institution defaulting. The crisis was generated by the financial system itself. This fact—that the defect was inherent in the system —contradicts the prevailing theory, which holds that financial markets tend toward equilibrium and that deviations from the equilibrium either occur in a random manner or are caused by some sudden external event to which markets have difficulty adjusting. The severity and amplitude of the crisis provides convincing evidence that there is something fundamentally wrong with this prevailing theory and with the approach to market regulation that has gone with it" (Soros 2008 p.63)

If Rodrik and Soros are right - and I believe they are - then there is a serious challenge ahead at the interface of mathematics and the social sciences. Because we live in a culture that has come to orient itself in a global economy with the help of mathematical metaphors - just as we use and need mathematical metaphors to orient ourselves in the worlds of nature and of technology.

In today's global culture, economic literacy begins with two related metaphors. The first has been introduced in ordinary language by Adam Smith and invokes the image of the invisible hand of the market. The second invokes the image of a rising supply curve intersecting a falling demand curve. Starting with the work of Walras (1874), this mathematical metaphor became the nucleus of crystallization for our current understanding of the global economy we live in. When trying to explain why a particular price - say the price of oil - is rising or falling, shifts in demand and/or supply are invoked routinely to describe how the invisible hand of the market operates.

"As in ordinary language, metaphors may be used in mathematics to explain a given phenomenon by associating it with another which is (or is considered to be) more familiar. It is this sense of familiarity, whether individual or collective, innate or acquired by education, which enables one to convince oneself that one has understood the phenomenon in question" (Aubin 1993, p.9). The current financial crisis is confronting us with the need for other mathematical metaphors to understand how markets work.

Of course, practical action often cannot wait for appropriate metaphors to emerge. Financial Times columnist John Kay (2008) approvingly quotes president elect Barack Obama who "told The New York Times: 'My own core economic theory is pragmatism.' In the present crisis, that is a pretty good place from which to begin."

At the juncture of a global financial crisis and a looming global recession, pragmatic action is widely seen to require the injection of huge amounts of additional money into the global economy as well as equally huge additions to effective demand by governmental deficit spending. While these measures seem indispensable to avoid a global economic crisis of historic proportions, they are also known to engender the risk of massive subsequent inflation as well as new speculative bubbles leading to even larger crises than the present one. Prevailing economic theory is of little help in assessing what are reasonable amounts of additional money and additional effective demand at what moment in time, or in deciding the directions in which those resources should be targeted. Generating mathematical metaphors that will be helpful for those purposes, then, becomes a key ingredient of a pragmatic approach to our economic worries.

Generating appropriate metaphors, however, is likely to take years and decades - in painful contrast to the speed at which pragmatic decisions must currently be taken. Still, generating such metaphors, organizing them in reasonably coherent theories, embodying them in practically meaningful computer models, is the challenge for research.

To meet this challenge, it is worth listening to Aubin (1993, p.9/10) at some length: "Contrary to popular opinion, mathematics is not simply a richer or more precise language. Mathematical reasoning is a separate faculty possessed by all human brains, just like the ability to compose or listen to music, to paint or look at paintings, to believe in and follow cultural or moral codes, etc. [...]

Naturally, the construction of mathematical metaphors requires the autonomous development of the discipline to provide theories which may be substituted for or associated with the phenomena to be explained. This is the domain of pure mathematics. The construction of the mathematical corpus obeys its own logic, like that of literature, music or art. [...]

That is not all. A mathematical metaphor associates a mathematical theory with another object. There are two ways of viewing this association. The first and best known way is to search for a theory in the mathematical corpus which corresponds as precisely as possible with a given phenomenon. This is the domain of applied mathematics, as it is usually understood. But the association is not always made in this way; the mathematician should not be simply a purveyor of formulae for the user. Other disciplines, notably physics, have guided mathematicians in their selection of problems from amongst the many arising and have prevented them from continually turning around in the same circle by presenting them with new challenges and encouraging them to be daring and question the ideas of their predecessors. These other disciplines may also provide mathematicians with metaphors, in that they may suggest concepts and arguments, hint at solutions and embody new modes of intuition. This is the domain of what one might call motivated mathematics.

Motivated mathematicians must possess a sound knowledge of another discipline and have an adequate arsenal of mathematical techniques at their fingertips together with the capacity to create new techniques (often similar to those they already know). In a constant, difficult and frustrating dialogue they must investigate whether the problem in question can be solved using the techniques which they have at hand or, if this is not the case, they must negotiate a deformation of the problem (a possible restructuring which often seemingly leads to the original model being forgotten) to produce an ad hoc theory which they sense will be useful later. They must convince their colleagues in the other disciplines that they need a very long period for learning and appreciation in order to grasp the language of a given theory, its foundations and main results and that the proof and application of the simplest, the most naive and the most attractive results may require theorems which may be given in a number of papers over several decades; in fact, one's comprehension of a mathematical theory is never complete."

Clearly, motivated mathematicians working on social science issues need transdisciplinary minded social scientists as partners. The author of the present paper is a social scientists writing in this perspective.

2 Coordination by Conventions

In the search for new mathematical metaphors to improve our understanding of the global economy, a conjecture of Soros provides a promising hint: "As a way of explaining financial markets, I propose an alternative paradigm that differs from the current one in two respects. First, financial markets do not reflect prevailing conditions accurately; they provide a picture that is always biased or distorted in one way or another. Second, the distorted views held by market participants and expressed in market prices can, under certain circumstances, affect the so-called fundamentals that market prices are supposed to reflect. This two-way circular connection between market prices and the underlying reality I call reflexivity.

While the two-way connection is present at all times, it is only occasionally, and in special circumstances, that it gives rise to financial crises. Usually markets correct their own mistakes, but occasionally there is a misconception or misinterpretation that finds a way to reinforce a trend that is already present in reality and by doing so it also reinforces itself. Such self- reinforcing processes may carry markets into far-from-equilibrium territory. Unless something happens to abort the reflexive interaction sooner, it may persist until the misconception becomes so glaring that it has to be recognized as such. When that happens the trend becomes unsustainable and when it is reversed the self-reinforcing process starts working in the opposite direction, causing a sharp downward movement." (Soros 2008, p.63).

The dynamics of the financial crisis that was triggered by the breakdown of the U.S. market for subprime mortgages fits well with this idea, as does the transition from this financial crisis to a global slowdown of economic growth. And the tools of multiscale analysis of dynamical systems hold promise to turn the basic intuition into powerful mathematical metaphors and models.

However, two related difficulties arise. First, Soros sticks to the image of a single equilibrium defined unambiguously at any moment in time by economic fundamentals. And second, he pictures economic agents as simply not smart enough to perceive that equilibrium adequately. But if this were the whole problem, then it is hard to see why anybody – governments, international institutions, or what have you – should be smart enough to do anything about the resulting crises.

Things get clearer if we look at the economy as a stochastic dynamical system with a variety of basins of attraction. The mechanisms of supply and demand work quite well within a given basin, but the selection of the basin itself confronts economic agents with a coordination problem of a different kind. As financial markets deal with contracts referring to future transactions, economic agents need to guess what basins of attraction the economy will evolve through in an undetermined future. When forming their guesses, they observe each other and develop a kind of herd behavior. The selection of a basin of attraction then

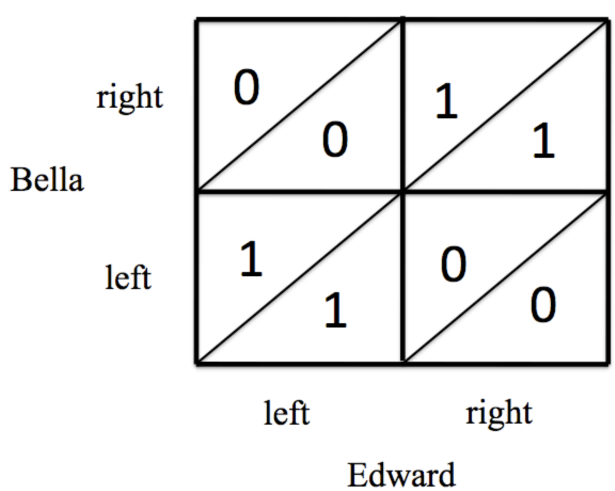

Figure 1: A simple coordination game

becomes a matter of convention. And when looking at conventions and their dynamics, a rich thread of mathematical metaphors becomes available.

One of the best examples to introduce the concept of a convention is the rule for driving on the right hand side or the left hand side of a road. This can be represented by a simple coordination game: let there be two players, called

Bella and Edward. They both have two strategies, called left and right. If they choose the same strategies their payoffs are 1, which is supposed to be good, if they choose different strategies, the payoffs are 0, supposed to be bad.

The resulting pattern is represented in figure 1. The same pattern is found in the game of matching pennies, and of course in many more.[2]

This example can be generalized to an abstract game theoretic setting. Let there be N players, each with a strategy set that is a subset of some real valued vector space. In the example above, it is sufficient to consider a set of two arbitrary numbers, one for left and one for right. Each strategy combination leads to some outcome. In our example, the outcomes may be smooth traffic and car crashes.

$$\begin{aligned} N \in I\!N &: \text{Number of players} \\ S_n \subset I\!R^{\zeta} &: \text{Set of strategies of player } n = 1,..,N \\ S = \prod_{i=1}^{N} S_n \subset R^{\zeta * n} &: \text{Set of strategy combinations} \\ s \in S &: \text{strategy profile} \\ s_n &: \text{strategy of player } n \text{ in profile } s \\ \Omega \subset I\!R^{\xi} &: \text{Set of outcomes} \\ \phi : S \to \Omega &: \text{Consequences of joint action} \end{aligned} \tag{1}$$

Each player has preferences over outcomes, and this leads to the usual definitions of Pareto superiority and Nash equilibria:

$$\begin{aligned} \succeq_n &: \text{Weak preferences of agent } n \\ &\quad \text{they form a total preorder on } \mathit{\Omega} \\ \succ_n &: \text{Strict preferences of agent } n \\ &\quad \omega \succ_n \omega' :\equiv \omega \succeq_n \omega' \wedge \neg\, \omega' \succeq_n \omega \\ \succ_P &: \text{Pareto superiority} \\ &\quad \omega \succ_p \omega' :\equiv \exists n(\omega \succ_n \omega') \wedge \neg\, \exists n(\omega' \succ_n \omega) \\ \sim_P &: \text{Pareto indifference} \\ &\quad \omega \sim_p \omega' :\equiv \neg\, \omega \succ_p \omega' \wedge \neg\, \omega' \succ_p \omega \\ Q \subset S &: \text{Set of Nash equilibria} \\ &\quad Q = \{s| \neg\, \exists n\, \exists s'\, \forall m \\ &\qquad\qquad (\phi(s') \succ_n \phi(s) \wedge \\ &\qquad\qquad (m \neq n \Rightarrow s_m = s'_m))\} \end{aligned}$$

In this setting, conditions for the existence and uniqueness of Nash equilibria and Pareto optima and for various relations between the two can be established

[2]In each of the four boxes of figure 1 the number above the diagonal indicates the payoff for Bella, playing the rows, and the number below the diagonal the payoff for Edward, playing the columns.

– as has been abundantly done in the literature. In particular, it has been show that it is very easy to construct games with many Nash equilibria as well as social dilemma games where Nash equilibria are not Pareto optimal.[3]

I now define a convention as a strategy profile s with the following two properties:

- it yields a Nash equilibrium
- there is at least one other strategy profile s' yielding a Nash equilibrium that is Pareto indifferent in comparison with the one given by s.

$$\begin{aligned} \Gamma \subset Q &: \text{Set of conventions} \\ \Gamma &= \{s|\ s \in Q \wedge \exists s'\ (s \neq s' \wedge s' \in Q \wedge \phi(s) \sim_p \phi(s'))\} \end{aligned} \tag{2}$$

Depending on what one wants to study, there may be reasons to modify this definition (e.g. one may drop the condition of Pareto indifference). But in order to address the problem of price formation on interdependent markets, the current version provides a good starting point for the argument to be developed in the following sections. With the game illustrated in figure 1, the existence of

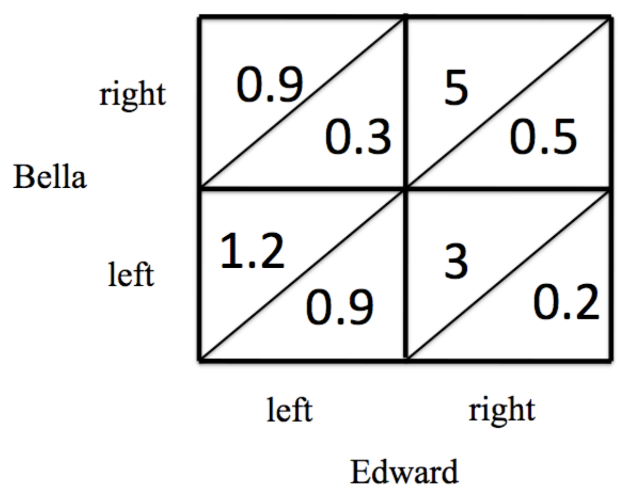

Figure 2: A less obvious coordination game

two conventions can be read from the obvious symmetry of the payoff matrix.

[3]A key existence and uniqueness proof was provided by the pioneering work of von Neumann and Morgenstern (1947); the pervasiveness of multi-equilibria games is know as the 'folk theorem' of game theory, because it has been established by so many authors; the literature on social dilemma games – including the famous prisoners dilemma – originated in 1950 out of work by Flood, Dresher, and Tucker (Poundstone 1992).

However, conventions arise in more general cases as well, as figure 2 shows. In the former case, both players are indifferent between the outcome resulting from the two conventions. In the latter case, things are different: while Bella prefers the convention leading to the upper right corner of the payoff matrix, Edward prefers the alternative possibility. The point of a convention is that it solves a coordination problem, but it may solve it in ways that put certain players at a disadvantage if compared with alternative conventions, while being to the advantage of other players. Clearly, this leads to the possibility of conflict about the choice of conventions.[4]

The case where the disadvantages of the former can be quantitatively compared with the advantages of the latter is mathematically advantageous, but by no means the rule in social systems. Even more interesting from a mathematical point of view, albeit not very frequent in social systems, is the case where disadvantages and advantages of different players not only can be compared quantitatively, but actually sum up to zero – the so-called zero-sum games.

With this notion of convention, one can study much more than choices of traffic lanes and games of matching pennies. In particular, it provides a root metaphor for the mathematical analysis of the global economy we live in (see e.g. the emphasis on coordination problems in Colander 2006).

3 The Dynamics of Conventions

At the end of the first chapter of their classic "Theory of Games and Economic Behaviour", von Neumann and Morgenstern (1947) wrote: "Our theory is thoroughly static. A dynamic theory would unquestionably be more complete and, therefore, preferable. But there is ample evidence from other branches of science that it is futile to try to build one as long as the static side is not thoroughly understood"

In the past decades, much progress has been made in the analysis of dynamic patterns in settings inspired by - although sometimes quite different from - the original game theoretical setting. Particularly relevant for our present purpose is work on the dynamics of conventions in networks of agents (e.g. Young 1993).

Consider a finite population of $N > 2$ agents engaged again and again in a game of the kind represented in figures 1 and 2. The strategy the agents choose depends on the past k (with k some odd integer – say 3) games they played. If their prevailing experience has been with players driving on the right hand side, that's what they will do, and conversely for the left hand side. For convenience, let one additional agent be the null player: if a regular (i.e. non-null) – player is matched with the null player, no game takes place, and the memory of the

[4]The term payoff is unfortunate, as the payoffs in matrices like those of figures 1 and 2 are utility indices representing preferences. If the payoffs of Bella are doubled and those of Edward cut by half, the structure of the game remains exactly the same. The widespread habit of simply adding the payoffs of all players to obtain a "value" of the outcome can be quite misleading.

regular player stays unchanged.

Writing R for an opponent driving on the right, L for an opponent driving on the left, the memory of each agent can be represented by a list like RRL or LRL. The state of the system is given by the list of all those lists, i.e. the memory content for each agent. The number of possible states is 8^N, all possible combinations of eight memory contents for each regular agent.

At each iteration, a matching of agents takes place such that each player except the null player is assigned exactly one partner. If this matching is given by some deterministic mechanism, one gets a dynamical system in discrete time with a finite state space. Depending on the initial state, it will converge to everybody driving on the right or on the left, or it will enter an endless cycle of agents alternating between the two strategies without ever reaching a final agreement on what side to choose.

Now let the matching depend on some random process. To take a simple case, all possible permutations such that any agent i (except the null player) is matched with exactly one agent $j \neq i$, might have the same probability. Call the probability distribution over all possible permutations P. The possibility that state ν of the system will change to state μ is determined once the matching pattern is given, and so the probability of that change, $\pi_{\nu,\mu}$, depends on the distribution P. As a result, we get a stochastic process that can be described by a $8^N * 8^N$ Markov matrix $\Pi = (\pi_{\nu,\mu})$.

The two conventions of driving on the right or on the left now constitute absorbing states of these Markov processes: if everybody has been driving on the right for two iterations, no permutation will lead any agent to drive on the left anymore. And each convention has its basin of attraction, i.e. a set of initial states for which the probability of ending end up in the convention is larger than the probability of not doing so.

"If, in addition, the players sometimes experiment or make mistakes, then society occasionally switches from one convention to another" (Young, 1993, p.57). Experiments and mistakes then lead to an evolutionary dynamics that can be described by a second, strictly positive Markov matrix Π_ϵ that perturbs the first.[5] The result of the perturbation then is $\tilde{\Pi} = (1-\epsilon) * \Pi + \epsilon * \Pi_\epsilon$, with ϵ a small positive number. The point is that in $\tilde{\Pi}$ transitions that are ruled out in Π become possible, although only with small probability (hence the ϵ).

Therefore, the two conventions do not constitute absorbing states anymore. There still are basins of attraction corresponding to each one of them: from any state in the basin of a convention, the probability of reaching it in finite time is larger than the probability of not doing so. But there is a small probability of jumping out of the basin, and even if everybody has played the convention twice, there still is some minute probability of leaving it.

Unless the evolutionary process represented by Π_ϵ has implausible symmetry properties, the probabilities of switching from one basin of attraction to another one are asymmetric, resulting in different expected values for the time of permanence in each basin. As a result, we get a multistable system with different

[5] The same behaviour results if some power of Π_ϵ is strictly positive, even if Π_ϵ is not.

conventions, some of which will be in place for longer times than other.

Clearly, the dynamics of conventions now depends critically on the structure of the social network represented by the matching probabilities P. Phenomena like the diffusion of behavioral patterns with some "contagious" quality can be studied in such a framework. Examples range from herd behavior on financial markets to criminal behavior in urban neighborhoods as well as to innovative behavior in economic regions.

4 Prices as Conventions

In the century stretching from the pioneering work of Walras (1874) to the celebrated theorem of Sonnenschein-Mantel-Debreu (Sonnenschein 1973), economists could honestly neglect the issue of conventions. There was no reason to flatly ignore them, but they were not deemed essential to understand the functioning of a market economy. With hindsight, the research program of this long stretch of scholarly work can be related to the framework introduced in equations (1) as follows.

Let the players be firms, households, and a fictitious player representing the "invisible hand" of the market (aka the "auctioneer"). The strategies of firms and households consist in statements of demanding and supplying various goods and services in various quantities, the strategies of the auctioneer consist in declaring a nonnegative price for each good and service. For reasons that will become apparent in a moment, the preferences of the auctioneer are assumed to aim at maximizing the absolute value of excess demand, i.e. the absolute value of the scalar product between the price vector and the differences between demand and supply.

Now assume each strategy set $S_n,\ n = 1, .., N$ to be compact and convex. Then S is compact and convex, too. Assume further that there is a continuous mapping ψ from S to S with the following property. Given a strategy profile s, for each player $s^* = \psi(s)$ indicates a unique strategy s^*_n that produces the outcome player n prefers among those available if the other players play the strategies they have in s. $\psi(s)$ is called the best reply to s. Clearly, a fixed point of ψ is a Nash equilibrium. Given the alleged preferences of the auctioneer, a Nash equilibrium is a situation in which he is unable to increase the absolute value of excess demand. This is the case if and only if excess demand is zero, i.e. supply and demand match. That is the point of defining the preferences the way we did. As ψ is a continuous endofunction on a compact convex set in $R^{\zeta * n}$, by Brouwer's fixed point theorem it has a fixed point.

This scheme of argument has been refined and expanded in many ways.[6] Perhaps the most important move was to consider a set-valued correspondence Ψ from S to $\mathcal{P}(S)$, the power set of S, thereby taking into account the possibility that there may be more than one best reply to a given profile s. Brouwer's

[6] Border (1985) still provides one of the best overviews.

theorem must - and can - then be generalized to what has become known as Kakutani's fixed point theorem.

This long effort was guided by the intuition that sooner or later it would be possible to show that a system of interdependent markets could be characterized by a single general equilibrium somehow akin to the familiar intersection of demand and supply curves on a single market. The Sonnenschein-Mantel-Debreu theorem put this intuition to rest by showing that with more than two markets, the standard assumptions accepted in the existence proofs for a general equilibrium are compatible with any number of general equilibria. Subsequent work has shown that the assumptions needed to guarantee uniqueness are way beyond anything remotely plausible - including, e.g., the requirement that all households have the same preferences.

Computer models are still built routinely in ways that do guarantee uniqueness - e.g. by assuming a single aggregated household so that the above requirement is trivially met. Modelers are usually not even aware of the moves by which they avoid the consequences of Sonnenschein-Mantel-Debreu, although these moves may play quite a role in blinding them for possibilities like the current financial crisis.

Recently, Gintis (2007) has made an extremely interesting proposal for addressing this problem by building on the kind of convention dynamics discussed in the previous section. He starts from the observation that economic transactions usually happen between two agents and that this bilateral trade often happens at a price that is slightly different from the one arising in a similar trade elsewhere - a cup of coffee costs less than a car, but not all coffeeshops charge exactly the same price. This leads him to treat prices as conventions enacted in bilateral trade, where the matching between agents is the result of a random process and where price-setting agents do not obey the convention strictly. Supply and demand then operate as expected in the neighborhood of a conventional pattern of prices, while the convention as such is selected by the stochastic dynamics of the evolutionary process combining the matching mechanism with departure from uniform prices. Conventional price patterns yield Nash equilibria of the system of interacting markets. If they were followed strictly by all agents, it would be all but impossible to get a plausible dynamics ensuring at least local stability. The twin random processes of agent matching and of evolving prices set individually by firms, however, lead to a process of convention dynamics like the one discussed above.

This is a major step towards an improved understanding of the economy we live in. It embeds the mathematical metaphors of the equilibrium tradition in a family of metaphors dealing with coordination through conventions and its dynamics. It moves economic modeling from highly aggregated equilibrium models towards multi-agent models with more complex dynamics.

Still, we are only at the beginning. As Bilancini and Petri (2008) show, the analysis proposed by Gintis (2007) presupposes a world where there is a unique capital good given in unchanging quantity - more akin to the elementary economic concept of land than to capital goods. Related to this feature, financial markets are of a hyper-simplified kind that rules out investment bubbles from the outset, because there simply is no investment. Even with these drastic simplifications, the precise relations between the dynamics of conventions in games and the dynamics of prices and quantities in interdependent markets still invite further analysis.

But any interesting research program is full of unresolved issues worth further investigating. Together with A.Mandel, S.Fürst, W.Lass, R.Klein, F.Meissner and others I am developing a computer model of the Gintis type roughly calibrated on the German economy (labelled Lagom modeG) where heterogeneous capital is possible and can be accumulated. This leads to additional conventions related to financial markets. E.g., central banks can be understood as following a Taylor rule (Taylor 1993) combining a specific set of conventions: for the target inflation rate, the so-called natural rate of unemployment, and the reaction speed to divergencies between target and actual values.

5 Outlook

Research on conventions has been greatly stimulated by Lewis (1969), who proposed a philosophy based on the concept of conventions. Still, the attempt to let conventions carry the weight of our whole cognitive fabric is neither necessary nor convincing (Ben-Menahem, 2006). The relations between various kinds of conventions, and then their relations to various kinds of habits, rules, norms, values, etc. are far from clear. And the interplay between various kinds of conventions and the various kinds of meaning that they may or may not convey is so rich that there is much to be lost and little to be gained by overemphasizing the role of conventions in human life.

That said, there seems to be little to be lost and much to be gained by embedding the prevailing understanding of how markets work in a dynamic analysis of economic conventions. This holds for the micro-economics of relative prices as well as for the macro-economics of inflation, employment, etc. It may hold for the dynamics of skills and technologies as well. As for practical relevance: dealing with global financial crises in a truly pragmatic, rather than haphazard, manner will require metaphors that can only emerge out of such research.

More generally speaking, there are strong synergies between such an analysis of markets and the study of non-market phenomena like the inertia, diffusion and occasional transformation of patterns of social behavior. Moving in this direction provides a fascinating challenge to mathematicians and social scientists interested in trans-disciplinary work.

References

Aubin, J.-P. (1993) Optima and Equilibria, An Introduction to Nonlinear Analysis. Springer, New York.

Ben-Menahem, Y. (2006) Conventionalism: From Poincaré to Quine. Cambridge University Press, Cambridge, U.K.

Bilancini, E. and Petri, F. (2008) The Dynamics of General Equilibrium: A Comment on Professor Gintis. Quaderni del dipartimento di economia politica, n. 538, Universitẹgli studi di Siena.

Border, K.C. (1985) Fixed Point Theorems with Applications to Economics and Game Theory. Cambridge UP, Cambridge, UK.

Colander, D. (2006) Post-Walrasian Macroeconomics: Beyond the Dynamic Stochastic General Equilibrium Model. Cambridge University Press, Cambridge, U.K.

Gintis, H. (2007) The Dynamics of General Equilibrium. Economic Journal, 117, 1280-1309.

Kay, J. (2008) Obama Is Right to Opt for Pragmatism. The Financial Times, Nov.18.

Kirman, A.P. (1992) Whom or what does the representative agent represent? Journal of Economic Perspectives, 6, 117-136.

Lewis, D.K. (1969) Convention: A Philosophical Study. Harvard University Press, Cambridge, Mass.

Poundstone, W. (1992) Prisoners Dilemma. Doubleday, New York.

Rodrik, D. (2008) Who Killed Wall Street? www.hks.harvard.edu/news-events/news/commentary/who-killed-wall-street (first published in the Korea Times of October 8).

Sonnenschein, H. (1973) Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions? Journal of Economic Theory, 6, 345-354.

Soros, G. (2008a) The Crisis & What to Do About It. The New York Review of Books, 55, December 4, 63-65.

Taylor, J. (1993): Discretion versus Policy Rules in Practice, Carnegie-Rochester Conference Series on Public Policy. 39, 195-214.

von Neumann, J. and Morgenstern, O. (1947) Theory of Games and Economic Behavior (2d – enhanced – edition), Wiley, New York.

Walras, L. (2003 / 1874) Elements of Pure Economics. Routledge, London.

Young, H. P. (1993) The Evolution of Conventions. Econometrica 61, 57-84.