Synchronization in the macro- and micro-economy: ideas and concepts

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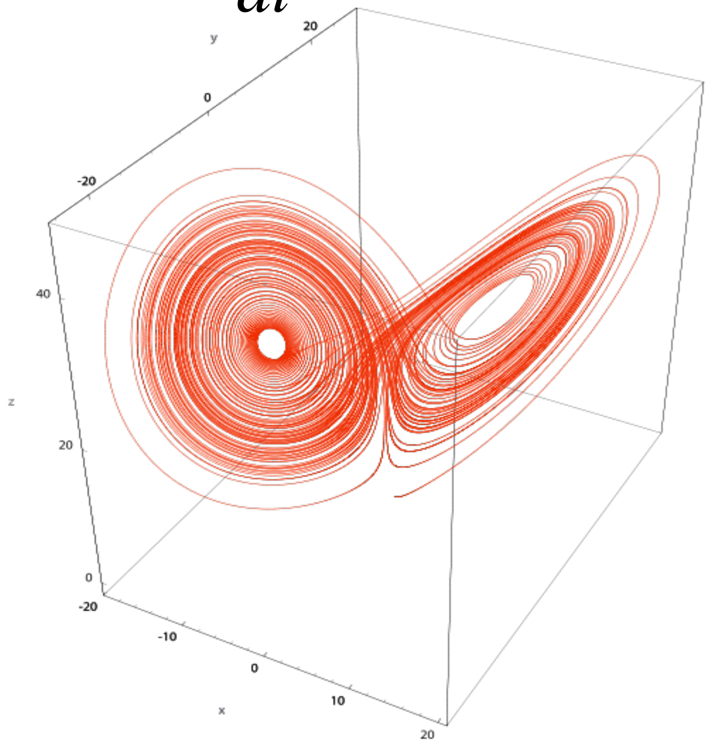
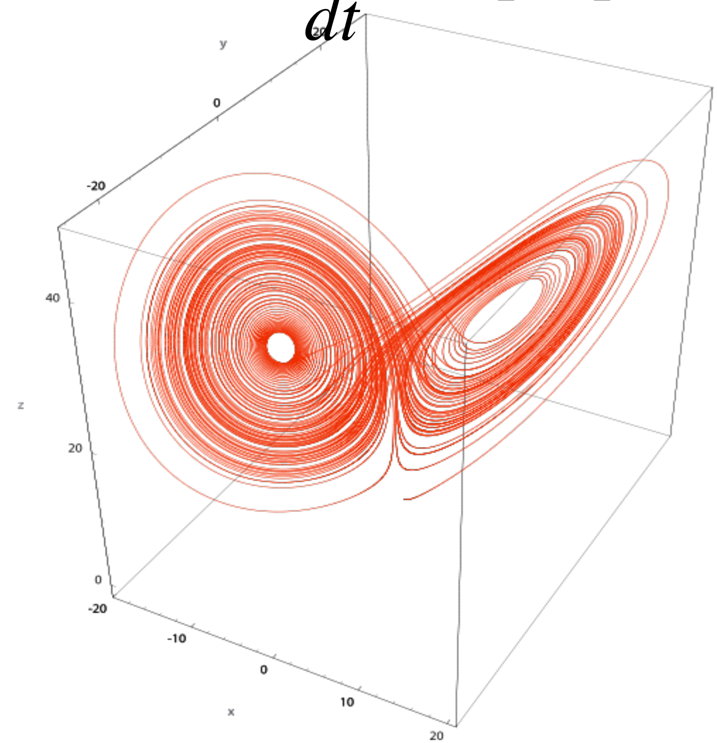
Outline

- Some general words about synchronization
- Phase synchronization, Kuramoto oscillators and consensus formation
- (Complete) synchronization on networks: business cycles in the macro-economy
- Summary

Introduction: Synchronization (1)

$$\frac{dx_1}{dt} = F_1(x_1) \qquad\qquad \frac{dx_2}{dt} = F_2(x_2)$$





What happens if the systems are coupled?

$$\frac{dx_1}{dt} = F_1(x_1) + s(x_2 \quad x_1) \qquad \frac{dx_2}{dt} = F_2(x_2) + s(x_1 \quad x_2)$$

Introduction: Synchronization (2)

- Various forms of synchronization can occur
  - Identical synchronization x1=x2
  - Generalized synchronization x1=G(x2)
  - Phase synchronization np1=mp2
  - Log synchronization x1(t)=x2(t+tau)
  - and some weaker forms of sync
- Dependence on the details of
  - the system F, the coupling strength s and the "architecture" of the coupling

Phase Sync: The Kuramoto System

- Dynamics of weakly coupled nearly identical limit cycle oscillators can often be approximated

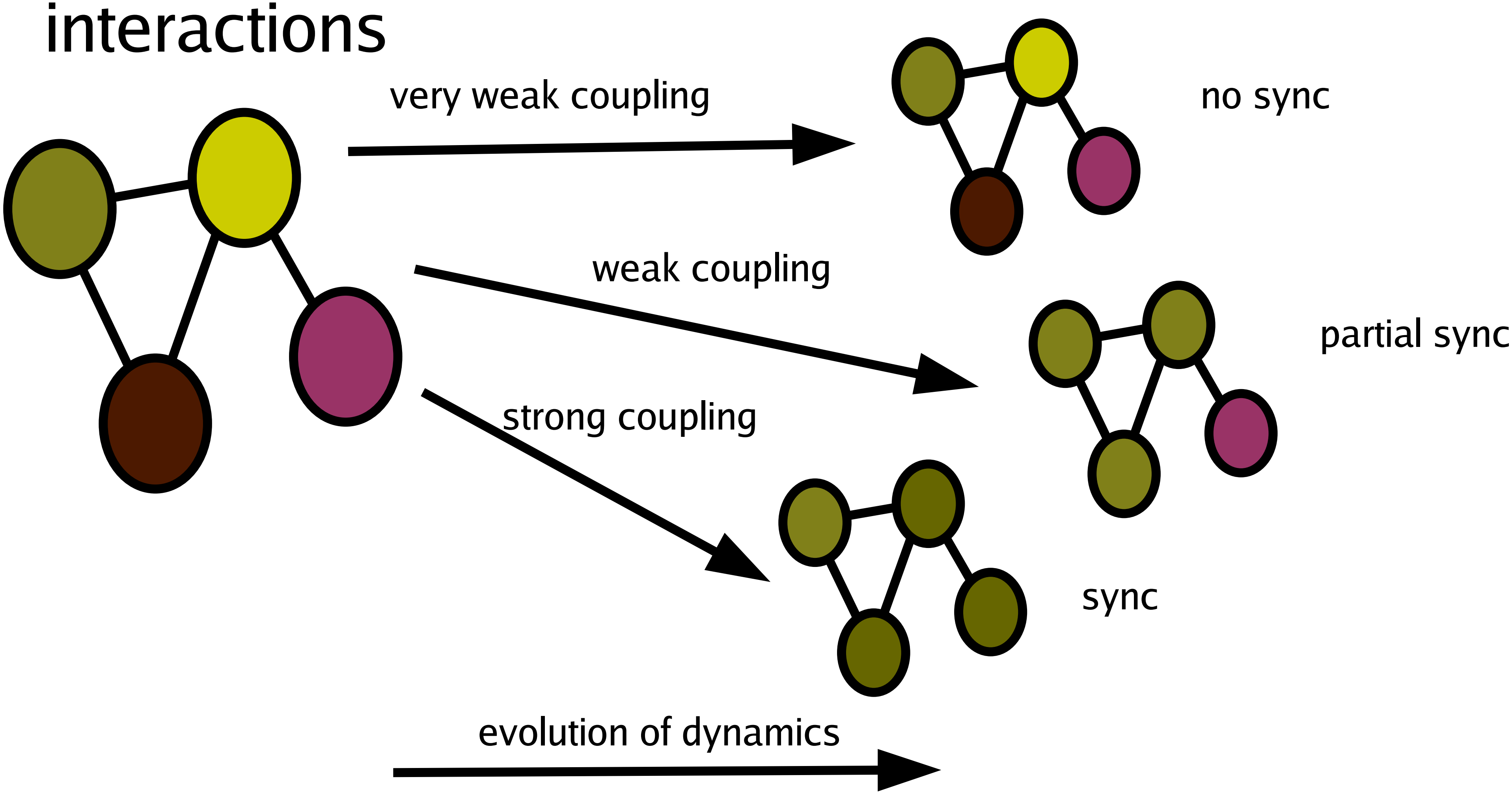
$$\frac{d\Theta_i}{dt} = \omega_i + \sigma \sum_j A_{ij} \sin(\Theta_j \quad \Theta_i)$$

- Conventional order parameter

$$r = 1/N \left| \sum_j \exp(i\,\Theta_j(t)) \right|$$

Kuramoto oscis and consensus formation

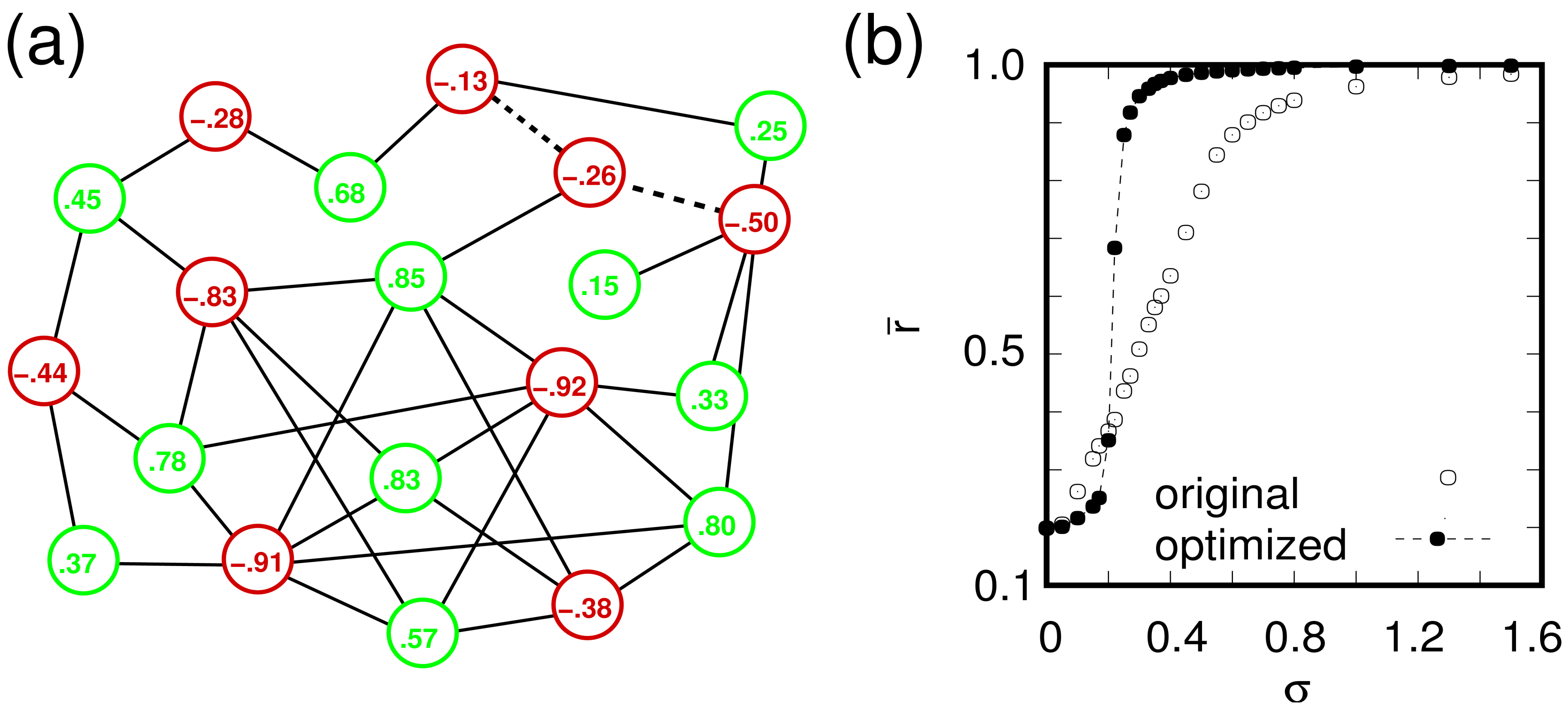
- Native frequencies: “native” opinion; Coupling network: social network; Coupling: social interactions



What is the influence of the system configuration on the final state?

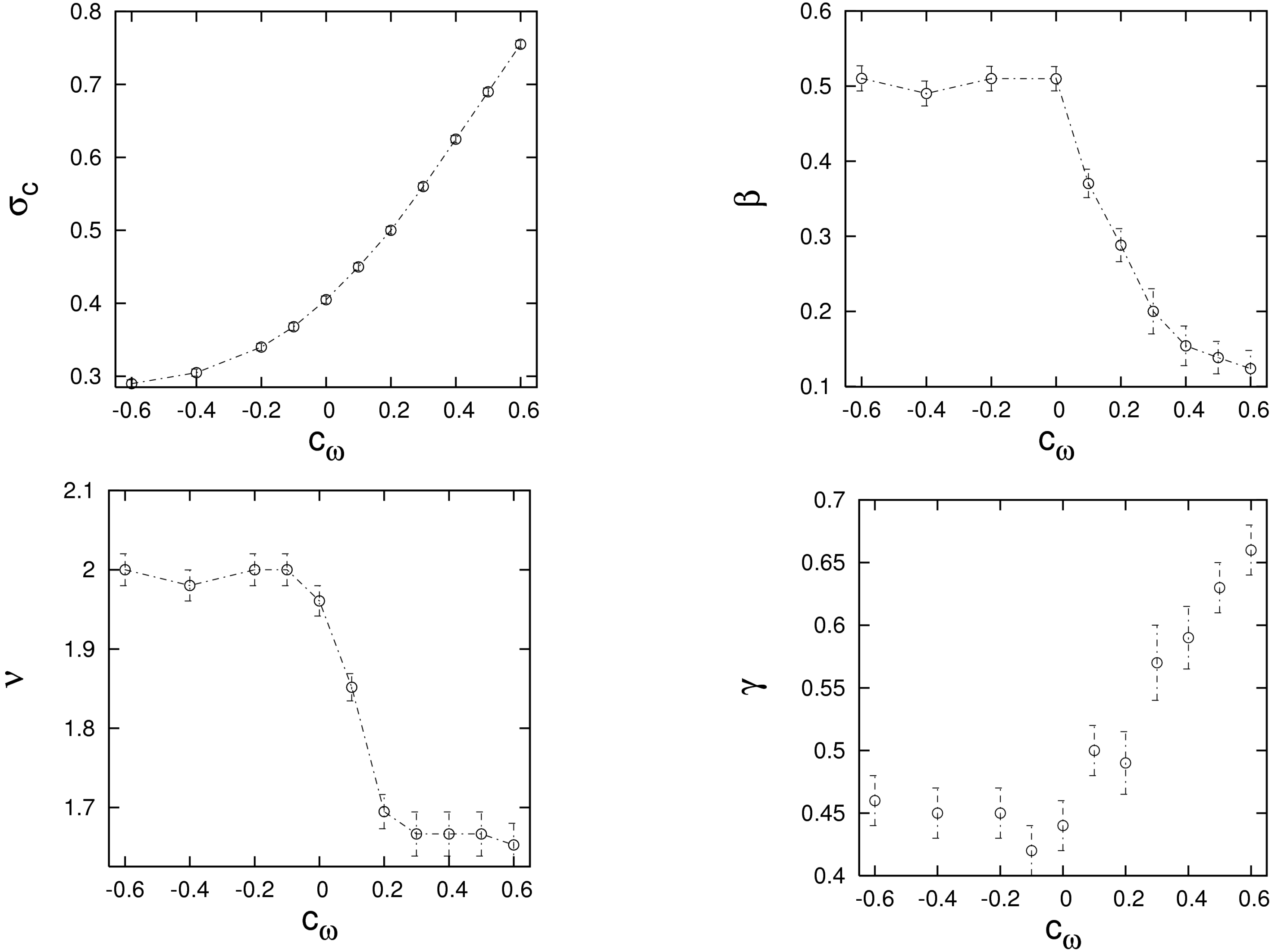
- Topology of the system configuration is determined by two factors:
  - network arrangement (who interacts with whom)
  - arrangement of “native” opinions (who has a tendency to believe in what)
- What implication do these factors have on:
  - the final state? (measured by r)
  - timescales to reach the final state

An example network



- Oscillator correlations matter! Define a correlation coefficient c_w between adjacent oscis.

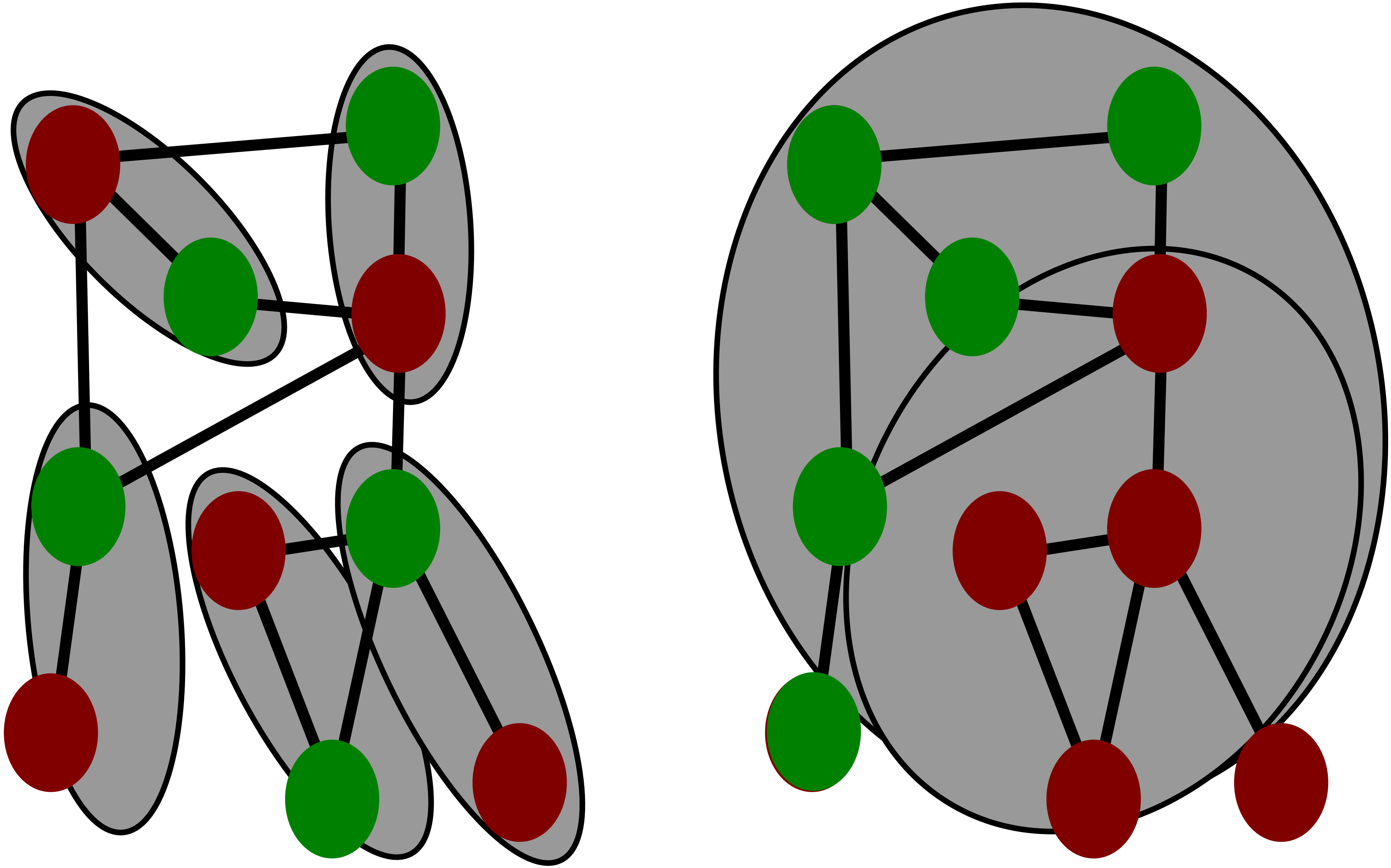
The synchronization transition on networks with corr. oscillators (1)



The synchronization transition on networks with corr. oscillators (2)

- Main result: The stronger the correlations between the “native”o pinions, the:
  - more coupling is required to achieve complete sync (and the more likely partial sync becomes)
  - the longer the timescales to the stationary state

Why? Some heuristics ...



Networks and Identical Sync

- Consider N chaotic oscillators with states $x_i(t) \in R^n$ and "native"d ynamics $F_i(x)$
- Weak coupling, e.g. via a network given by an adjacency matrix A:
- $$\frac{dx_i}{dt} = F_i(x) + \sigma \sum_j A_{ij} H(x_j \quad x_i)$$
- Depending on coupling strength and coupling topology synchronization of the states is possible

Applications: business cycles?

- Many processes in the economy seem to fit this framework
  - Actors (=nodes) can be at the firm or country level
  - Cyclical dynamics at many levels: inventory cycles, business cycles, etc.
  - Actors interact: trade, capital allocation, technology
- No sync: no cyclical dynamics at macrolevel (world, country, etc.)
- Sync: Cycles and oscillations at macrolevel

AIM

- Gain some insight under which conditions synchronization is expected to occur in “realistic”s ituations
- Focus now on network structure: Which features in network topology facilitate sync?

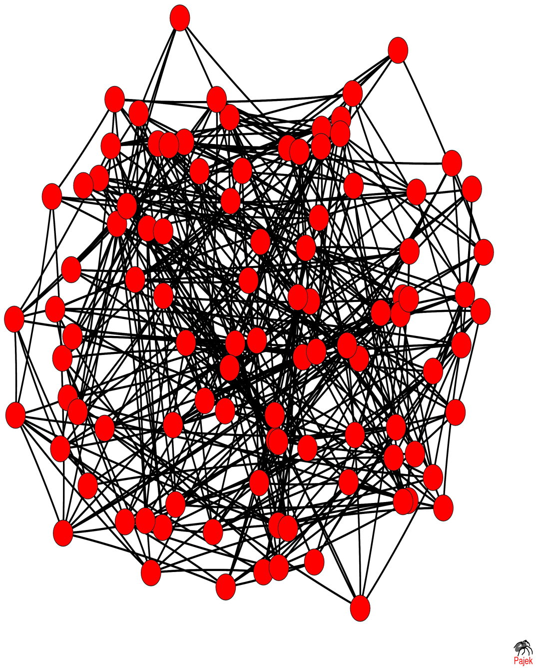
The MSF approach

- Understanding synchronization requirements via stability of fully synchronized state
  - identical oscillators (i.e. same “native” dynamics $F_i = F$ all i)
- Maximum Lyapunov exponent (=”master stability function”) often with a bounded negative region such that linear stability of synchronized state requires:

$$\sigma \lambda_i \in (\alpha_{1,} \alpha_2)$$

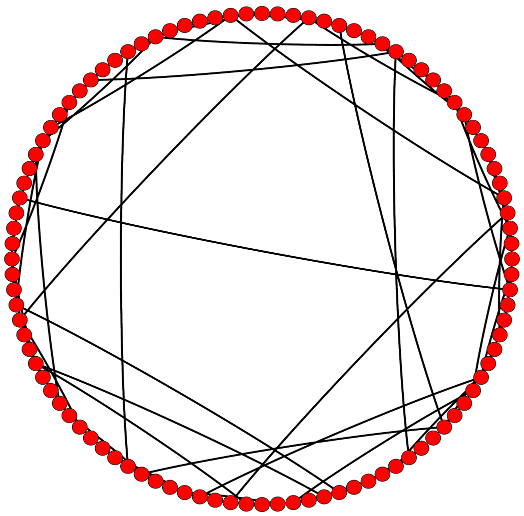
Some results

constrained number of links,
no spatial constraints



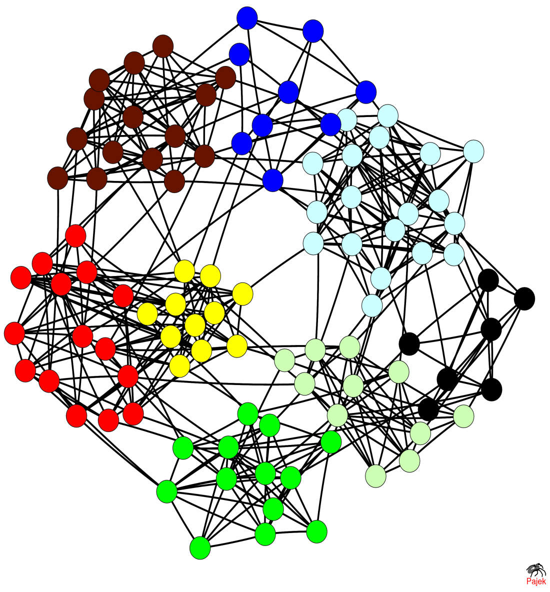
homogeneous,
small
no short cycles
(dissortative)

constrained number of links,
constraints on wire,
nodes with fixed locations



(relative) hubs as centres for communication,
small,
some short cycles
power law distribution of link lengths

constrained number of links,
constraints on wire,
nodes with flexible locations



modular arrangement in space and network
small,
many short cycles

Summary

- Considered two applications of sync problems in economic models
  - at the micro-level: consensus formation, clustering of “native” opinions one important determiner of transition to sync and timescales
  - at the macro-level: spatial constraints on the network arrangement lead to a variety of network arrangements in space that determine a network's “synchronizability”